

**University of Dhaka**  
**Department of Applied Mathematics**  
**Four year B.S. Honours Program**  
**Session: 2020-2021 (1<sup>st</sup> year) to 2023-2024 (4<sup>th</sup> year)**

**Degree Requirement: Successful completion of 139 credits**

<b>Major Courses</b>		<b>133 credits</b>
<b>Minor Courses</b>		<b>6 credits</b>
Theory Courses	116 credits	
Lab Courses	12 credits	
Project and Presentation	3 credits	
Viva Voce	8 credits	
<b>Total</b>		<b>139 credits</b>

**(Minor Subjects: Physics)**

**Year-wise Class-Load**

<b>First Year</b>		<b>32 credits</b>
Major Courses (Theory)	23 credits	
Minor Courses (Theory)	4 credits	
Math Lab	3 credits	
Viva Voce	2 credits	
<b>Second Year</b>		<b>33 credits</b>
Major Courses (Theory)	26 credits	
Minor Courses (Theory)	2 credits	
Math Lab	3 credits	
Viva Voce	2 credits	
<b>Third Year</b>		<b>35 credits</b>
Major Courses (Theory)	30 credits	
Math Lab	3 credits	
Viva Voce	2 credits	
<b>Fourth Year</b>		<b>39 credits</b>
Major Courses (Theory)	31 credits	
Math Lab	3 credits	
Project and Presentation	3 credits	
Viva Voce	2 credits	

## List of Courses

### **First Year** **32 credits**

AMTH 101	Fundamentals of Mathematics with Modeling	3 credits
AMTH 102	Applied Calculus	4 credits
AMTH 103	Coordinate and Vector Geometry	3 credits
AMTH 104	Applied Linear Algebra	3 credits
AMTH 105	Principles of Economics	3 credits
AMTH 106	FORTRAN Programming	3 credits
AMTH 107	Basic Statistics and Probability	4 credits
AMTH 150	Math Lab I (Mathematica)	3 credits
AMTH 199	Viva Voce	2 credits
PM 111	Mechanics and Waves	2 credits
PM 122	Electricity and Magnetism	2 credits

### **Second Year** **33 credits**

AMTH 201	Mathematical Analysis	3 credits
AMTH 202	Multivariate and Vector Calculus	4 credits
AMTH 203	Ordinary Differential Equations with Modeling	3 credits
AMTH 204	Advanced Linear Algebra	3 credits
AMTH 205	Numerical Methods I	3 credits
AMTH 206	Discrete Mathematics	3 credits
AMTH 207	Programming in C ++	3 credits
AMTH 208	Mathematical Statistics	4 credits
AMTH 250	Math Lab II (Fortran)	3 credits
AMTH 299	Viva Voce	2 credits
PM 224	Thermodynamics	2 credits

### **Third Year** **35 credits**

AMTH 301	Complex Variables and Fourier Analysis	3 credits
AMTH 302	Theory of Numbers and Groups	3 credits
AMTH 303	Partial Differential and Integral Equations	4 credits
AMTH 304	Mathematical Methods	4 credits
AMTH 305	Numerical Methods II	3 credits
AMTH 306	Mechanics	3 credits
AMTH 307	Hydrodynamics	3 credits
AMTH 308	Introduction to Financial Mathematics	3 credits
AMTH 309	Optimization Techniques	4 credits
AMTH 350	Math Lab III (Matlab)	3 credits
AMTH 399	Viva Voce	2 credits

### **Fourth Year** **39 credits**

AMTH 401	Applied Analysis	3 credits
AMTH 402	Fluid Dynamics	3 credits
AMTH 403	Physical Meteorology	3 credits
AMTH 404	Elementary Hydrology	3 credits
AMTH 405	Differential Geometry and Tensor Analysis	4 credits
AMTH 406	Asymptotic Analysis and Perturbation Methods	3 credits
AMTH 407	Stochastic Calculus	3 credits

**The courses from AMTH 408 - 413 and AMTH 430 will be offered as per the decision of the academic committee. Among the offered courses three courses will be chosen by the students.**

AMTH 408	Econometrics	3 credits
AMTH 409	Actuarial Mathematics	3 credits
AMTH 410	Heat Transfer	3 credits
AMTH 411	Modern Astronomy	3 credits
AMTH 412	Quantum Theory and Special Relativity	3 credits
AMTH 413	Mathematical Modeling in Biology and Physiology	3 credits
AMTH 430	Special Topics	3 credits
AMTH 450	Math Lab IV (Application Software)	3 credits
AMTH 460	Project and Presentation	3 credits
AMTH 499	Viva Voce	2 credits

## Detailed Syllabi

### AMTH 101: Fundamentals of Mathematics with Modeling

3 credits

#### Introduction and Specific Objectives

This course aims to provide learning of fundamental concepts of mathematics which are essential for mathematical thinking. The course includes concepts and theories such as elementary set theory, graph and relation, logic, number systems, sequences, series, inequalities, polynomials, voting system and apportionment. A unique emphasis on applied problems throughout the course utilizes each new technique and develops the conceptual aspects of algebra.

#### Learning Outcomes

On successful completion of this course students will be able to

- to understand set theory in detail and apply this concept to solve real world problems
- to understand mathematical concepts and definitions of functions and relations
- to apply precise, logical reasoning to problem solving
- to understand number systems, sequences, series and inequalities
- to apply the knowledge of this course to solve problems in business/ natural science/ economics.

#### Course Contents

1. **Set Theory:** Set concepts, subsets, Venn Diagrams & Set operations, Venn Diagrams with three sets & verification of equality of sets, Applications of sets in the Management, Natural & Social Sciences.
2. **Graph and Functions:** Define and identify relations & functions, Cartesian product of sets, Equivalence relations, notations for functions, Graphs of relation & functions, visualizing domain and range, composite functions, inverses, inverse & one-to-one functions, onto functions, finding formulas for inverses, restricting a domain, applications of functions.
3. **Logic:** Statements and Logical Connectives, Truth Tables for Negation, Conjunction, and Disjunction, Truth Tables for the Conditional and Biconditional, Equivalent Statements, Symbolic arguments, Euler Diagrams and Syllogistic Arguments, Switching Circuits, applications and models.
4. **Real number system, Sequences and Series:** Number Theory, The Integers, The Rational Numbers, The Irrational Numbers and the Real Number System, Real Numbers and Their Properties, Arithmetic and Geometric Sequences & Series, Fibonacci Sequence, Applications and problem solving situations in business/ natural Science/ Economics.
5. **Inequalities:** Basic inequalities, Inequalities involving means, powers; inequalities of Cauchy, Chebyshev and applications.
6. **Polynomials:** Polynomial Functions and Modeling (The leading term test, finding zeros of polynomial functions, polynomial models) , Polynomial Division; The Remainder and Factor Theorems ( Division and Factors, The remainder theorem, Synthetic division, finding factors of polynomials), Theorems about zeros of polynomial functions (The fundamental theorem of Algebra, finding polynomials with given zeros, zeros of Polynomial Functions with real coefficients, rational coefficient, integer coefficients and the rational zeros theorem, Descartes' rule of signs), Polynomial models and applications.
7. **Voting and Apportionment:** Voting Methods, Flaws of Voting, Apportionment Methods, Flaws of the Apportionment Methods.

**Evaluation:** Incourse Assessment and Attendance 30 marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

#### References

1. Angel, Abbott and Runde, A survey of Mathematics with applications, 10th edition, Pearson.
2. Blitzer, Thinking Mathematically, 6th edition, Pearson.
3. Beecher, Penna and Bittinger, College Algebra, 5th edition, Pearson.
4. S. Lipschutz, Set Theory, Schaum's Outline Series.
5. S. Barnard & J. M. Child, Higher Algebra.

**Introduction and Specific Objectives**

The course aims to provide a firm foundation in the concepts and techniques of the calculus, including basic functions and graphs and their properties, curve sketching, limits, continuity, differentiation, relative extrema and applications, Taylor Series. This course also introduces the student to learn Integral Calculus, to the techniques of integration and to some of the applications of integration to physical problems.

**Learning Outcomes**

At the end of the course the students will be able to interpret a function from an algebraic, numerical, graphical and verbal perspective and extract information relevant to the phenomenon modeled by the function. They will be able to understand the concept of limit and continuity of a function at a point graphically and algebraically using appropriate techniques. The students will be able to interpret the derivative of a function at a point as the instantaneous rate of change and as the slope of the tangent line, they will also learn to compute the value of the derivative at a point algebraically using the (limit) definition. They will interpret the value of the first and second derivatives as measures of increase and concavity of a function. They will also understand the consequences of Rolle's theorem and the Mean Value theorem for differentiable functions.

The students will be able to interpret the definite integral geometrically as the area under a curve and construct a definite integral as the limit of a Riemann sum. They will learn to understand differentiation and anti-differentiation as inverse operations (Fundamental Theorem of Calculus, part 1) and will learn to evaluate integrals using techniques of integration.

The students will be able to understand the different types of improper integrals and solve them. The students will also learn how to calculate the area between curves, volumes of solids of revolution, surface area, arc length using integration.

**Course Contents****Part A: Differential Calculus**

1. **Functions and their graphs:** polynomial and rational functions, logarithmic and exponential functions, trigonometric functions and their inverses, hyperbolic functions and their inverses, composition of functions.
2. **Limit and Continuity of Functions:** Definition. Basic limit theorems, limit at infinity and infinite limits. Continuous functions. Properties of continuous functions on closed and bounded intervals.
3. **Differentiability and related theorems:** Tangent lines and rates of change. Definition of derivative. One-sided derivatives. Rules of differentiation. Successive differentiation. Leibnitz theorem. Related rates. Linear approximations and differentials. Rolle's theorem, Lagrange's and Cauchy's mean value theorems. Extrema of functions, problems involving maxima and minima. Concavity and points of inflection. Indeterminate forms. L'Hospital's rule.
4. **Power series expansion:** Taylor's theorem with general form of the remainder; Lagrange's and Cauchy's forms of the remainder. Taylor's series. Maclaurin series.
5. **Applications:** Physical, Biological, Social Sciences, Business and Industry.

**Part B: Integral Calculus**

6. **Integrals:** Antiderivatives and indefinite integrals. Techniques of integration. Definite integration using antiderivatives. Definite integration using Riemann sums. Fundamental theorems of calculus. Basic properties of integration. Integration by reduction. Improper integrals. Improper integrals of different kinds. Gamma and Beta functions.
7. **Graphing in polar coordinates:** Tangents to polar curves. Area and arc length in polar coordinates.
8. **Applications of integration:** Plane areas. Solids of revolution. Volumes by cylindrical shells. Volumes by cross-sections. Arc length and surface of revolution.

**Evaluation:** Incourse Assessment and Attendance 30 marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, four from each part, of which **five** are to be answered, taking at least **two** questions from each part.

### References

1. H. Anton, I. C. Bivens, S. Davis, Calculus.
2. E.W. Swokowski, Calculus.
3. James Stewart, Calculus: Early Transcendentals.
4. Deborah Hughes-Hallett, Applied Calculus.
5. Stefan Waner and Steven Costenoble, Applied Calculus.

## AMTH 103: Coordinate and Vector Geometry

**3 credits**

### Introduction and Specific Objectives

The Geometry course includes an in-depth analysis of plane, solid, and coordinate geometry as they relate to both abstract mathematical concepts as well as real-world problem situations. Topics include logic and proof, parallel lines and polygons, perimeter and area analysis, volume and surface area analysis, similarity and congruence, trigonometry, and analytic geometry. Emphasis will be placed on developing critical thinking skills as they relate to logical reasoning and argument. Students will be required to use different technological tools and manipulatives to discover and explain much of the course content.

In the Euclidean Geometry course students will deepen their understanding of: (1) geometric relationships in a plane in space, (2) meaning and nature of proofs, (3) deductive proof in both mathematical and non-mathematical situations, (4) integrate geometry with arithmetic and vector algebra.

### Learning Outcomes

Students that successfully complete this course will be able to

- sketch graphs of and discuss relevant features of lines, circles, and other conic sections and determine equations of curves when given information that determines the curves
- perform translations and rotations of the coordinate axes to eliminate certain terms from equations and use the polar coordinate system, relate it to the rectangular coordinate system, and graph equations using polar coordinates
- compute the distance between points, the distance from a point to a line, and the distance from a point to a plane in the three-dimensional coordinate system
- sketch and describe regions in space and perform algebraic operations with vectors in two and three dimensions computing dot and cross product of vectors, finding scalar and vector projections of a vector onto another, determining if vectors are parallel and orthogonal, etc
- find equations of lines and planes in space and identify and describe quadratic surfaces.

### Course Contents

1. **Coordinates, Equations and Graphs:** The rectangular coordinate system: The coordinate plane, Test for symmetry and their applications, Equations of lines and Circles and their graphs, Applications, extensions and changes of both rectangular and polar coordinates.
2. **Transformations:** Translations of axes, Equation of a curve in a translated system, Graphing a translated conic, Rotation of axes, analyzing an equation using a rotation (identify and sketch), Identifying conics without rotation (use discriminant).
3. **Conic sections:** Standard equations of parabolas, ellipses and hyperbolas and their properties. Solve applied problems involving parabolas, ellipses and hyperbolas. Polar Equations of conics, Finding a polar equation of an orbit, Application to describe a closed orbit of a satellite around the sun (Earth or Moon).
4. **Pair of straight lines:** Ideas of pair of lines, Equation of a pair of lines, lines passing through origin, angle between the lines, general equation of second degree, Equation of the angle of bisectors, Homogenous equation of second degree.

5. **Three dimensional coordinate system:** Rectangular coordinate system in 3-space, Octants, Direction cosines and direction angles, direction ratios, angle between two lines, projection on a line, Applications to Human biomechanics, Genome expression profiles, Antigenic cartography, species, vaccine design and vaccination.
6. **Parametric equations of lines:** Vector equation of a line, parametric form of equation of a line, symmetric equations, intersection of two line (parametric form), Different types of lines (perpendicular, parallel and skew), shortest distance and equations.
7. **Plane in three space:** Equation of a plane (vector and rectangle equations), line of intersection of two planes, distance between two skew lines, point of intersection, intersection of a line with other curves, planes and surfaces, Finding distance between two parallel lines, Angle between two intersecting planes, Distance between a point and a plane.
8. **Vectors in space:** Geometric vectors, vectors in a coordinate plane, position vector, sum and difference of vectors, magnitude, unit vectors, graphs of the sum and difference. Dot product and Cross product: physical interpretation of the dot product (applications and extensions), orthogonal vectors, component and projection of a vector on another, cross product of basis vectors, right hand rule, physical interpretation of the cross product (applications and extensions) Areas, scalar triple product, volume of a parallelepiped, coplanar vectors.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Khosh Mohammad, Analytic Geometry and Vector Analysis.
2. H. Anton et al, Calculus with Analytic Geometry.
3. D. G. Zill and J. M. Dewar, Pre-calculus with calculus previews.
4. Michael Sullivan, Pre-calculus.
5. Howard Anton, Iri Bivens and S. Davis, Calculus Early Transcendental.

## AMTH 104: Applied Linear Algebra

**3 credits**

### Introduction and Specific Objectives

This course provides an introduction to the concepts and techniques of Linear Algebra. It will focus on matrix and vector methods for studying systems of linear equations, with an emphasis on concrete calculations and applications. Specific topics to be covered include matrices, Gaussian elimination, vector spaces, orthogonality, determinants, inner products, linear transformations, eigenvalue problems, and Markov processes.

This major core course aims at introducing students to the fundamental concepts of linear algebra culminating in abstract vector spaces and linear transformations. This covers systems of linear equations, matrices, and some basic concepts of the theory of vector spaces in the concrete setting of real linear  $n$ -space,  $R^n$ . The subject material is of vital importance in all fields of mathematics and science.

### Learning Outcomes

On successful completion of this course students will be able to

- solve systems of linear equations by using Gaussian elimination to reduce the augmented matrix to row echelon form or to reduced row echelon form
- understand the basic ideas of vector algebra: linear dependence and independence and spanning and apply the basic techniques of matrix algebra, including finding the inverse of an invertible matrix using Gauss-Jordan elimination
- know how to find the row space, column space and null space of a matrix, to find bases for these subspaces and be familiar with the concepts of dimension of a subspace and the rank and nullity of a matrix, and to understand the relationship of these concepts to associated systems of linear equations

- compute the eigenvalues and eigenvectors of a square matrix using the characteristic polynomial and will know how to diagonalize a matrix when this is possible
- understand the general notions of a vector space over a field and of a subspace, linear independence, dependence, spanning sets, basis and dimension of a general subspace and find the change-of-basis matrix with respect to two bases of a vector space
- use the notion of a linear transformation, its matrix with respect to bases of the domain and the codomain, its range and kernel, and its rank and nullity and the relationship between them;
- know how to find the eigenvalues and eigenvectors of linear operators.

### Course Contents

1. **Matrices and Determinants:** Review of matrix and determinants. Different types of matrices, elementary row and column operations and row-reduced echelon matrices, rank. Block matrices, Invertible matrices, matrix exponentials. Applications of Matrices and determinants: Leontief Input-Output Models, Constructing Curves and Surfaces through Specified Points, Markov Chains, Graph Theory.
2. **System of Linear Equations:** Linear equations. System of linear equations (homogeneous and non-homogeneous) and their solutions. Application of Matrices and determinants for solving system of linear equations. Gaussian and Gauss-Jordan eliminations. Applications of linear systems: Network Analysis (Traffic Flow), Electrical Circuits, Polynomial Interpolation, Bacterial growth etc.
3. **Vector Spaces:** Review of geometric vectors in  $\mathbb{R}^n$  and  $\mathbb{C}^n$ . Vector space and subspace. Sum and direct sum of subspaces. Linear independence of vectors, basis and dimension of vector spaces. Row spaces, column spaces and null spaces, rank and nullity of a matrix. Application of Vector Spaces: Solution spaces of systems of linear equations.
4. **Linear Transformations:** Linear transformations. Kernel and image of a linear transformation and their properties. Matrix representation of linear transformations. Change of bases, Transition matrix. Applications of Linear Transformations: Geometry of Matrix Operators on  $\mathbb{R}^2$ .
5. **Eigenvalues and Eigenvectors:** Definition of Eigenvalues and eigenvectors, diagonalization. Cayley-Hamilton theorem. Applications of Eigenvalues and Eigenvectors: Markov Chains, Age-Specific Population Growth.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. H. Anton, and C. Rorres, Linear Algebra with Applications.
2. S. Lipshutz, Linear Algebra, Schaum's Outline Series.
3. Brestscher, Linear Algebra with Applications
4. D. Lay, Linear Algebra with Applications
5. G. Strang, Linear Algebra with Applications.

## AMTH 105: Principles of Economics

**3 credits**

### Introduction and Specific Objectives

This course is an introduction to the fundamental principles of economics. Main objective of the course is to teach foundational concepts in economic theory to prepare student of applied mathematics so that they can apply their knowledge in development of economic theory. The course will cover essential topics such as demand and supply; consumer and producer behavior, taxes and subsidies; market structure and competition; inflation and unemployment, national income etc.

### Learning Outcomes

Upon successful completion of the course the student will be able to

- understand basic economic problems and how different economic system solve those problems, consumer and producer behavior and the nature of demand and supply



- understand how households (demand) and businesses (supply) interact in various market structures to determine price and quantity of a good produced
- know about different kinds of markets and how they function, and the welfare outcomes of consumers and producers
- apply economic reasoning to individual and firm behavior
- understand the basics of national income accounting, the roles of fiscal and monetary policy in fighting recessions and inflation
- apply economic reasoning to understand the operation of an economy.

### Course Contents

1. **Basic Concepts:** Definition and scope of economics, basic economic problems and their sources, choice, tradeoff and opportunity cost, economic systems - command economy, market economy and mixed economy; microeconomics and macroeconomics.
2. **Demand and supply:** definition, factors influencing them, demand and supply schedules & curves, law of demand, market demand and market supply, movements along and shifts in demand curve, shifts in supply curve, market equilibrium: price theory in the market, its implications, effects of a shift in demand or supply on equilibrium position, special cases.
3. **Elasticity:** Elasticity of demand and supply - concepts, definitions and problems associated with calculations, price elasticity, income elasticity and cross elasticity of demand, consumer's expenditure pattern and total revenue in relation to elasticity of demand, computation of elasticity from demand function and family budget data.
4. **Consumer Behavior and Utility:** basic concepts, ordinal and cardinal measurements of utility, consumer's preference ordering. Total utility and marginal utility, relationship between them, law of diminishing marginal utility, equimarginal principle. Substitution and income effects and the law of demand. Slutsky equation, computation of elasticity from Slutsky equation. consumer's surplus and its applications.
5. **The indifference Curve Analysis:** Indifference curve analysis as an improvement over Marshallian analysis, consumer's indifference curve: properties, rate of commodity substitution. The equilibrium position of tangency: consumer's equilibrium, effects of income and price change on equilibrium.
6. **Production and Cost:** Concept of a Production Function, factors of production-fixed and variable, Total, Average and Marginal Product, the Law of Diminishing Returns, Returns to Scale; Costs: Fixed and Variable Cost, Total, Average and Marginal cost, Short Run and Long Run Costs.
7. **Decision of firm and Revenue:** Isoquants, Isocosts and the Least Cost Combination, Total, Average and Marginal Revenue; Equating Marginal Revenue with Marginal Cost, Market Structure, Perfectly Competitive Markets, Monopoly, Oligopoly (Game Theory), Monopolistic Competition.
8. **Macroeconomics:** key concept of macroeconomics: GDP, GNP, Real vs. Nominal GDP, Price Deflators; Saving, consumption, investment; National income analysis; Inflation, Unemployment; Fiscal and monetary policy, Investment Decision, Cost benefit analysis, NPV, IRR, Payback period.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Samuelson, P. A. and W. D. Nordhaus (2004). Economics, 18th Edition, McGraw-Hill/Irwin.
2. Pindyck, R. S. and D. L. Rubinfeld (2012). Microeconomics, 8th Edition, Pearson Education.
3. Mankiw, G. N. (2012). Macroeconomics, 8th Edition, Worth Publishers, Inc.

## **AMTH 106: Fortran Programming**

**3 credits**

### **Introduction and Specific Objectives**

Fortran was developed for general scientific computing and is a very popular language for this purpose. This course provides an introduction, the structure and contents of the Fortran programming language. It will provide the students with enough knowledge to write Fortran programs and the students will gain general experience that can usually be applied when using any programming language. The main objective of this course is to expose students to algorithmic-problem solving and to develop fundamental skills in Fortran programming, with emphasis on a transparent and disciplined programming style, code modularity and reusability of the components.

### **Learning Outcomes**

After successful completion of this course, the students will be able to understand the basic components of the digital computer, the basic characteristics of several operating systems, several number systems, and the conversion of numbers from one system to others. They will know the evolution, development and standard solving techniques of Fortran programming language. The students will also learn several loops, decision statements, several external and internal procedures of Fortran programming in detail. They will be able to use arrays, allocate memory for arrays and use files efficiently in Fortran programming language.

### **Course Contents**

1. **Introduction to Computation:** Introduction to Digital Computers; Operating Systems; Programming and Problem Solving.
2. **Number System:** Binary; Octal; Decimal and Hexadecimal number systems. Conversion of numbers from one system to others.
3. **Fundamentals of Computer Programming:** Programming basics; High-level programming languages; Introduction to FORTRAN; Fortran Evolution; Basic differences among FORTRAN 77, Fortran 90, Fortran 2003 and Fortran 2008; Problem-Solving Techniques: Flowcharts and Algorithms.
4. **Programming in Fortran:** Syntax and semantics; Constants; and Variables; INTEGER, REAL, COMPLEX, LOGICAL and CHARACTER Data Types; Arithmetic, Relational and Logical Operations; Operator Precedence; Single and Mixed Mode Arithmetic; Expressions and Assignment Statements; Fortran Input/Output.
5. **Control Constructs and Arrays:** All IF Constructs; Nested and Named IF Constructs; SELECT CASE Construct; DO Loops; WHILE Loops; DO WHILE Loops: Named and Nested Loops; Declarations and Construction of Arrays; Memory allocation for Arrays; Memory ALLOCATABLE, Problems solving using Arrays.
6. **Programming Units:** Intrinsic Functions, Types of Programming Units; Main Program; External Procedures; Internal Procedures; FUNCTION Subprograms; SUBROUTINES; Subprogram for recursion.
7. **Use of Files:** Necessity of using files; opening and closing of files; reading from files; writing into files; Construction and implementation of Fortran programs for solving mathematical problems using Files.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### **References**

1. Stephen J. Chapman, Fortran for Scientists and Engineers, 4<sup>th</sup> Edition, 2018, McGraw Hill.
2. Michael Metcalf, John Reid, Malcolm Cohen, Modern Fortran explained, 6<sup>th</sup> Edition, 2011, Oxford University Press.
3. Walter S. Brainerd, Guide to Fortran 2008 Programming, 2009, Springer.

## **AMTH 107: Basic Statistics and Probability**

**4 credits**

### **Introduction and Specific Objectives**

This course is intended to provide the basic foundations of statistics with applications in real life. The class will cover topics on descriptive statistics, correlation, regression, probability, and probability distributions for both continuous and discrete random variables. The students will discuss the theory

and how to apply and use the theory for real life problem-solving and inquiry. A central objective is to provide students with hands on experience in using the statistical theory and methods to perform the different statistical analyses and to interpret results.

### Learning Outcomes

After successfully completing this course, a student will be able to: Demonstrate the ability to apply fundamental concepts in exploratory data analysis, Construct and analyze graphical displays to summarize data, Compute and interpret measures of center and spread of data, Calculate, interpret and communicate the correlation coefficient and simple linear regression model, Utilize basic concepts of probability including independence and conditional probability to calculate, interpret and communicate event probabilities both for discrete and continuous random variables, Determine the appropriate probability distribution based on experiment conditions and assumptions.

### Course Contents

#### Part A: Basic Statistics

1. **Definition and Scope:** Definitions of statistics- past and present, its nature and characteristics, Methods of statistics, Scope and application of statistics, Abuse of statistics. Sources of statistical data, Primary and secondary sources of data.
2. **Processing and Presentation of Data:** Measurement scales; Variables, Attributes, Classification, Characteristic and basis of classification, Array formation. Tabulation, Different types of tables, Frequency distribution. Graphical presentation of data, Details of different types of graphs and charts with their relative merits and demerits.
3. **Characteristics of Statistical Data:** Measures of Location, Dispersion, Skewness, Kurtosis and their properties, Moments. Schematic plots.
4. **Correlation and Regression:** Bivariate data. Scattered diagram, Simple correlation, Rank correlation, Correlation ratio, Intra-class and bi-serial correlation, Multiple and partial correlations. Simple regression analysis, Principles of least squares, Lines of best fit, Standard error of estimators of regression coefficients, their properties, and their applications.

#### Part B: Probability

5. **Basic concepts of probability:** Meaning of probability, Scope of probability, Definition of probability, Different types of probability definitions: classical, axiomatic empirical and subjective. Difference between probability and possibility. Laws of probability, Conditional probability, Theorem of total probabilities.
6. **Bayes theorem:** Uses and importance of Bayes theorem in statistics.
7. **Random variables:** Discrete and continuous random variables, probability mass function, probability density function, Distribution function, function of random variable and its distribution, joint distribution, marginal and conditional distributions, independence of random variables, Mathematical expectation, expectations of sum and products of random variables. Moments and moment generating functions, Cumulants and cumulant generating functions, Relation between moments and cumulants.
8. **Probability Distributions:** Detail study of Binomial, Poisson & Normal distributions.

**Evaluation:** Incourse Assessment and Attendance 30 marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, four from each part, of which **five** are to be answered, taking at least **two** questions from each part.

### References

1. Islam, M.N. (2006). An Introduction to Statistics & Probability, Book World, Dhaka.
2. Sheldon Ross; Introductory Statistics, Third Edition.
3. Larson, R. and Farber, B. (2003), Elementary Statistics.
4. Yule and Kendall, M.G. An Introduction to the Theory of Statistics, Charles Griffin, London.
5. Mosteller, Rourke Thomas, Probability with Statistical Application.

**AMTH 150: Math Lab I (Mathematica)****3 credits**

Introduction to the computer algebra package MATHEMATICA. Problem solving in concurrent courses (e.g. Calculus, Linear Algebra and Geometry) using MATHEMATICA.

**Lab Assignments:** There are at least 07 lab assignments.

**Evaluation:** Internal Assessment (Laboratory works) 40 Marks. Final examination (Lab: 3 hours) 60 Marks.

**AMTH 199: Viva Voce****2 credits**

Viva voce on courses taught in First Year.

**Introduction and Specific Objectives**

Mathematics has become an indispensable tool in many areas including the physical sciences, engineering and computer sciences as well as economics and management science and mathematical analysis is one of the main pillars of mathematics. The study of mathematical analysis has great value for the students who wish to go beyond the routine manipulations of formulas to solve standard problems, for ability to think deductively and analyze complicated examples to modifying and extending concepts to new contexts.

The objective of the course is to introduce the basic ideas of real analysis with particular emphasis on metric spaces.

**Learning Outcomes**

- A sound knowledge of the sets of real numbers, boundary of a set, supremum, infimum, limit, interior, exterior of a set and when a set close or open.
- Understanding the idea of infinite sequences of real numbers and their convergences.
- Working knowledge of the convergence of infinite series of real numbers.
- Understanding the concept of limit, continuity, differentiability of real valued functions.
- Be familiar with the Riemann integral in  $\mathbb{R}$  and  $\mathbb{R}^n$  and able to calculate the values of integrals.
- Ability to deal with various examples of metric spaces, open and closed sets in metric spaces.
- Implementations of the convergence of infinite sequence of metric in many real life problems.

**Course Contents**

1. **Real number system:** Supremum and infimum of a set, cluster (limit) points; the completeness axiom, Dedekind's theorem and Bolzano-Weierstrass theorem (No proof), Open and closed sets, interior, exterior and boundary of a set, cluster point and derived set.
2. **Infinite sequences:** Sequences of real number, Convergence, Monotone sequences, subsequences, Cauchy's general principle of convergence, some important sequences.
3. **Infinite series of real numbers:** convergence and absolute convergence, Tests for convergence; Power series, Uniform convergence, differentiation and integration of power series.
4. **Limit, continuity and differentiability:** Limit, continuity and differentiability of functions, properties, Intermediate value theorem (no proof), Uniform continuity, Differentiation in  $\mathbb{R}^n$ , Implicit and inverse function theorems (Statements and verifications, and applications only, no proof).
5. **The Riemann integral:** definitions via Riemann's sums and Darboux's sums, Darboux's theorem, (equivalence of the two definitions) Necessary and sufficient conditions for integrability and integration in  $\mathbb{R}^n$ .
6. **Metric Spaces:** Definition and examples.  $\epsilon$ - neighborhood, open and closed sets in metric spaces, Interior, exterior and boundary of a set.. Cluster points of sets in metric spaces, Derived set, closure of a set. Bounded sets, Equivalent metrics.
7. **Infinite sequences in metric spaces:** Infinite sequences in metric spaces and their convergence, Cauchy sequences, Complete metric spaces, Continuity and uniform continuity of functions on metric spaces, Sequences and series of functions and their convergence.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

**References**

1. W. Rudin, Principles of Mathematical Analysis.
2. R. G. Bartle, Introduction to Real Analysis.
3. W F Trench, Introduction to Real Analysis.
4. Malik Arora, Mathematical Analysis.
5. Fatema Chowdhury and Munibur Rahman Chowdhury, Essentials of Real Analysis.

**Introduction and Specific Objectives**

This course takes calculus from the two dimensional world of single variable functions into the three dimensional world, and beyond, of multivariable functions. Students explore the following topics: vector geometry and the analytic geometry of lines, planes and surfaces; calculus of curves in two or three dimensions, including arc length and curvature; calculus of scalar-valued functions of several variables, including the gradient, directional derivatives and the Chain Rule; Lagrange multipliers and optimization problems; double integrals in rectangular and polar coordinates; triple integrals in rectangular, cylindrical and spherical coordinates; calculus of vector fields including line integrals, curl and divergence, fundamental theorem for line integrals and Green's theorem.

In this course our goal will be to master the techniques of calculus in two and three variables, such as finding and analyzing critical points, and evaluating multiple integrals. More broadly, we will attempt to develop an underlying geometric intuition that will allow us to understand the problems on a qualitative (as well as quantitative) level. For the most part our focus will be more on the practical than the theoretical, in that we will not spend a lot of time on rigorous proofs of theorems. We will spend a bit of time discussing applications but will be more concerned with ensuring that we've developed the necessary mathematical toolkit to understand such problems whenever they are encountered outside of this course.

**Learning Outcomes**

The students will be able to

- perform operations with vectors in two and three dimensional space and apply to analytic geometry
- differentiate and integrate vector-valued functions and apply calculus to motion problems in two and three dimensional space
- determine the limits, derivatives, gradients, and integrals of multivariate functions.
- solve problems in multiple integration using rectangular, cylindrical, and spherical coordinate systems
- select and apply appropriate models and techniques to define and evaluate line and surface integrals; these techniques will include but are not limited to Green's, Divergence, and Stoke's theorems.

**Course Contents****Part A: Differential Calculus**

1. **Vector-valued functions:** Introduction to Vector-Valued Functions, Calculus of Vector-Valued Functions, Tangent lines to graphs of vector-valued functions. Arc length from vector view point. Arc length parameterization.
2. **Curvature:** Unit Tangent, Normal and Binormal Vectors, Curvature of plane and space curves: Curvature from intrinsic, Cartesian, Parametric and Polar equations. Radius of curvature. Centre of curvature.
3. **Partial Differentiation:** Functions of several variables, Graphs of functions of two variables, Limits and continuity, Partial derivatives, Differentiability, linearization and differentials. The Chain rule. Partial derivatives with constrained variables, Directional Derivatives and Gradients, Tangent Planes and Normal Vectors, Extrema of functions of several variables, Lagrange multipliers. Taylor's formula for functions of two variables.

**Part B: Integral Calculus**

4. **Double Integrals:** Double Integrals over Nonrectangular Regions, Double Integrals in Polar Coordinates, Surface Area; Parametric Surfaces and Applications of Double Integrals.
5. **Triple integrals:** Volume as a triple integral, Triple Integrals in Cylindrical and Spherical Coordinates, Centers of Gravity Using Multiple Integrals and Applications of Triple Integrals, Change of Variables in Multiple Integrals; Jacobians.

6. **Topics in vector calculus:** Vector Fields, Gradient, Divergence, curl and their physical meanings  
Line Integrals, Green's Theorem, Surface Integrals, The Divergence Theorem, Stokes' Theorem,  
Applications of Surface Integrals; Flux.

**Evaluation:** Incourse Assessment and Attendance 30 marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, four from each part, of which **five** are to be answered, taking at least **two** questions from each part.

### References

1. H. Anton, Irl Bivens, Stephen Davis, Calculus: Early Transcendentals, 10th Edition.
2. J. Stewart, Calculus: Early Transcendentals, 6th Edition.
3. E. Swokowski, Calculus with Analytic Geometry.
4. R. T. Smith and R. B. Minton, Calculus: Early Transcendental Functions 4th Edition.

## AMTH 203: Ordinary Differential Equations with Modeling

**3 credits**

### Introduction and Specific Objectives

This course provides an introduction to ordinary differential equations and modeling with them. Formulation and solution of ordinary differential equations play a vital role in modeling physical and biological phenomena. This course focuses on how to find the analytical solutions of linear and nonlinear ODE, in particular, of first and second order using different methods.

The objective of this course is to introduce the basics of ordinary differential equations and terminologies. It focuses on the ability of the students to solve different types of ordinary differential equations analytically using well known techniques. Exploring the utility of ordinary differential equations in modeling physical and biological systems.

### Learning Outcomes

The students will be able to

- know the basic idea about differential equations, order, degree, classifications and modeling approach, initial value problems and autonomous differential equations
- formulate differential equations by removing arbitrary constants from algebraic relations and draw solutions curves using direction field
- find whether an initial value problem (IVP) has solution and whether the solution is unique by using the Existence and Uniqueness theorem
- classify first-order DE's as separable, homogeneous, linear, exact, Bernoulli's and solve them using appropriate methods
- know about higher order, mostly second order ODE's and their classifications such as Homogeneous and Nonhomogeneous
- solve them using reduction of order, method of undetermined coefficients, variation of parameters
- learn about Cauchy Euler equations and their solution process; several linear and nonlinear models using higher order ODE's, systems of linear differential equations using matrices, and applications of linear systems
- know about autonomous systems, stability and linearization of systems of nonlinear differential equation and some well-known mathematical models.

### Course Contents

1. **Introduction to Differential Equations:** Definition of Differential Equation, Order and Degree; Classification of Differential Equations; Formulation; Modeling Approach, Models and Initial Value Problems, Solution Curves without a solution: Direction fields, Autonomous first order DEs. The Modeling Process: Differential Systems.
2. **First-Order Differential Equations:** Existence and Uniqueness theorem (without proof), Solution of First-order DE's: Separable, Homogeneous, Linear, Exact, Solutions by substitutions, Linear models, Nonlinear models. Modeling with systems of first order DEs: Population models, Models of growth and decay, Acceleration velocity models: Motion of a falling body,

Compartmental analysis, heating and cooling of buildings, Newtonian mechanics, Electrical circuits.

3. **Higher-Order Differential Equations:** Homogeneous and Nonhomogeneous equations, Reduction of order, Homogeneous linear equations with constant coefficients, Undetermined coefficients, Variation of parameters, Cauchy Euler equations, Mass spring oscillator, Coupled Spring/Mass systems: Free damped motion, free undamped motion, Driven motion, Series circuit Analogue. Electrical Networks and Mechanical Systems, Linear models: BVP, Nonlinear models.
4. **Systems of Linear Differential Equations:** Matrix form of a linear system, Homogeneous and Nonhomogeneous linear systems, Second order systems and Mechanical applications. Metapopulations, Natural killer cells and Immunity, Transport of Environmental pollutants, Solution by Diagonalization.
5. **Systems of Nonlinear Differential Equations:** Chemical Kinetics: The Fundamental Theorem, Autonomous systems, Stability of linear systems, Ecological models: Predators and competitors, linearization and local stability.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Robert L. Borrelli and Courtney S. Coleman, Differential equations: A Modeling Perspective.
2. D. G. Zill and Warren S. Wright, Differential Equations with Boundary-Value Problems.
3. C. Henry Edwards, David E. Penney and David T. Calvis, Differential Equations and Boundary Value Problems: Computing and Modeling.
4. Nagle, Saff and Snider, Fundamentals of Differential Equations and Boundary Value Problems.

## AMTH 204: Advanced Linear Algebra

**3 credits**

### Introduction and Specific Objectives

The purpose of this course is to introduce the basic notions of linear algebra and to prove rigorously the main results of the subject. In this course the students will be familiar with the theory and computation of the Jordan canonical form of matrices and linear maps, bilinear forms, quadratic forms.

### Learning Outcomes

At the end of the course, students will be able to

- illustrate the advanced concepts of linear algebra
- reconstruct elementary proofs
- apply techniques from the course to various problems
- find the orthogonal and orthonormal bases
- synthesize major results of the course to give a 'big picture' of the material and its potential applications.

### Course Contents

1. **Similar Matrices:** Canonical forms of matrices, Similar matrices, Symmetric, orthogonal and Hermitian matrices.
2. **Linear Functional and Dual Space:** Linear functional and the dual space, Dual basis, Second dual space, Annihilators, Transpose of a linear transformation.
3. **Inner Product Space:** Inner products, Norms and inner product of vectors in  $R^n$  and  $C^n$ , Inner product spaces, Orthogonality and Gram-Schmidt process, orthonormal sets, Orthogonal complement, Linear functional and adjoints, Positive operators, Unitary operators and normal operators.
4. **Bilinear, Quadratic and Hermitian Forms:** Matrix form of transformations, Symmetric and skew symmetric bilinear forms, Canonical forms, Reduction form, Index and signature of real



quadratic form, Definite and semi-definite forms, Hermitian forms, Principal minors and factorable forms.

5. **Applications:** Solving problems in Mathematics, Physics, Social and Applied sciences.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Howard Anton and Chris Rorres, Elementary Linear Algebra Applications Version.
2. W. Greub, Linear Algebra.
3. Bernard Kolman, Linear Algebra.
4. WK Nicholson, Introduction to Abstract Algebra.

## AMTH 205: Numerical Methods I

3 credits

### Introduction and Specific Objectives

It may be considered to be a preparatory course in numerical analysis. While mathematical in nature, emphasis is also given to programming techniques for numerical methods. Introduction and application of numerical methods to the solution of physical and engineering problems. Techniques include iterative solution techniques, methods of solving systems of equations, and numerical integration and differentiation.

The goal is to cover a wide range of numerical methods to obtain an approximate solution of problems of physics where an exact solution is not available. A broad knowledge is often decisive to choose the right method when developing a new code.

### Learning Outcomes

At the conclusion of the course, students should be able to

- find numerical approximations to the roots of an equation by Newton method, Bisection Method, Secant Method, etc
- find numerical solution to a system of linear equations by Gaussian Elimination, Jacobi and Gauss-Seidel Iterative methods
- demonstrate the use of interpolation methods to find intermediate values for any given set of points
- apply several methods of numerical integration, including Romberg integration.

### Course Contents

1. **Preliminaries of Computing:** Basic concepts, Floating point arithmetic, Types of errors and their computation, Convergence.
2. **Numerical solution of non-linear and transcendental equations:** Bisection method, Method of false position. Fixed point iteration, Newton-Raphson method, Iterative method and Error Analysis.
3. **Interpolation and polynomial approximation:** Polynomial interpolation theory, Finite differences and their table, Taylor polynomials, Newton's Interpolation, Lagrange polynomial, Divided differences, Extrapolation.
4. **Numerical Differentiation and Integration:** Numerical differentiation, Richardson's extrapolation, Elements of Numerical Integration, Trapezoidal, Simpson's, Weddle's etc., Adaptive quadrature method, Romberg's integration.
5. **Numerical Solutions of linear systems:** Direct methods for solving linear systems, Gaussian elimination and backward substitution, pivoting strategies, numerical factorizations, Iterative methods: Jacobi method, Gauss Seidel method, SOR method and their convergence analysis.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. R.L. Burden and J. D. Faires – Numerical Analysis.
2. K. Atkinson – Introduction to Numerical Analysis
3. M. A. Celia and W. G. Gray – Numerical Methods for Differential Equations.
4. L.W. Johson & R. D. Riess, Numerical Analysis.

## AMTH 206: Discrete Mathematics

3 credits

### Introduction and Specific Objectives

This course is an introduction to the study of Discrete Mathematics, a branch of contemporary mathematics that develops reasoning and problem-solving abilities, with an emphasis on proof. Topics include logic, Boolean algebra, mathematical reasoning and proof, combinatorics and graph theory. The subject enhances one's ability to reason and ability to present a coherent and mathematically accurate argument. This course is intended for students of Applied Mathematics capable of and interested in progressing through the concepts of discrete mathematics in more depth and at an accelerated rate. The objectives of this course are to develop logical thinking with the emphasis of proving statements correctly and the correctness of an argument, to solve the circuit designing problems using Boolean algebra, and to develop skills to solve problems using graph theory. The main objective of this course is to provide basic ideas to identify and apply concepts of logic, Boolean algebra, proof techniques, combinatorics, graphs and trees.

### Learning Outcomes

After successful completion of this course, the students will be able to understand logical arguments and logical constructs, have a better understanding of logic and mathematical proofs and apply Boolean Algebra to construct gates and to minimize the circuits. The learners will understand the basics of Induction, Recursion, Recurrence relations and Generating functions, and be able to apply the methods from these topics in solving problems. The students will be able to understand the terminologies, definitions, concepts and methods of graphs and trees. They will have complete knowledge to solve realistic problems using the graphs and/or trees.

### Course Contents

1. **Logic and Mathematical Proofs:** Propositional logic and Equivalences; Rules of Inferences and Quantifiers; Various Quantified Statements; Methods of proof.
2. **Boolean Algebra:** Boolean Functions; Representing Boolean Functions; Logic Gates; Minimization of Circuits using Karnaugh maps.
3. **Induction and Recursion:** Mathematical induction; Well ordering; Recursive Definitions.
4. **Combinatorics:** The principle of Inclusion and Exclusion; Pigeonhole Principle. Recurrence relations; Applications to computer operations; Solving Linear Homogeneous and Nonhomogeneous Recurrence Relations; Generating Functions.
5. **Graph Theory and Applications:** Graphs; Graph Terminology; Special Types of Graphs; Representing graphs; Adjacency Matrices; Incidence Matrices; Graph Isomorphism; Paths; Circuits; Eulerian and Hamiltonian Paths; Shortest-Path problems; Dijkstra's Algorithm; Traveling Salesperson Problem, Planar Graphs.
6. **Trees:** Tree Terminology; Properties of Trees; Spanning Trees; Minimum Spanning Trees; Algorithms (Prim's and Kruskal's) for Minimum Spanning Trees and their comparison.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. K. H. Rosen, Discrete Mathematics and its Applications.
2. RP Grimaldi and BV Ramana, Discrete and Combinatorial Mathematics.
3. Bernard Kolman, Robert C. Busby, Sharon Cutler Ross, Discrete Mathematical Structures, Pearson Education.

**Introduction and Specific Objectives**

Programming skill represents a generic problem solving ability, and is considered essential for any science background student. The course is designed to provide complete knowledge of C++ language. Students will be able to develop logics which will help them to create programs and applications in C++. Also by learning the basic programming constructs they can easily switch over to any other language in future. The objective of this course is to impart adequate knowledge on the need of programming languages and problem solving techniques, to develop programming skills using the fundamentals and basics of C++ Language, to enable effective usage of arrays, structures, functions and pointers. It provides in-depth coverage of object-oriented programming principles and techniques. Topics include classes, overloading, data abstraction, information hiding, encapsulation, inheritance, polymorphism etc.

**Learning Outcomes**

The students will be able to understand

- basic idea of programming language and object oriented programming language, properties of object oriented programming language, how C++ improves C with object-oriented features, syntax and semantics of C++ programming language
- different data types, conditional logics, different arithmetic, relational and basic I/O operations. writing programs with different types of loops and to write programs using different types of arrays and Strings
- different types of functions, difference between call by value and call by reference, recursion. They will understand about code reusability with the help of various user defined functions.
- the basics of structures, pointers and various types of file operations
- design C++ programs with objects, classes, constructors, destructors, function overloading and operator overloading
- inheritance and virtual functions implement dynamic binding with polymorphism and how inheritance promote code reuse in C++.

**Course Contents**

1. **Basic Concepts:** Introduction to Computer Programming, Problem Solving Techniques, Programming Style, Debugging and Testing, Documentation.
2. **Object Oriented Programming Concepts:** Object Oriented Programming Overview, Encapsulation, Inheritance and Polymorphism. Object Oriented vs. Procedural Programming, Basics of Object Oriented Programming Language.
3. **Data Types, Conditional Logics and Operators:** Basic I/O, Data Types, Conditional Logics such as If, If-Else, Switch. Arithmetic, Relational, Logical and Bitwise Operators, Precedence and Associativity, Arithmetic Expression Evaluation.
4. **Loops, Arrays and Strings:** Looping Basic, Necessity of Loops, While Loop, For Loop, Do While Loop, Nested Loop. Basics of Array, Accessing through Indices, Accessing using Loops, Two Dimensional Arrays. Basics, I/O Operations using String, Basic String Operations.
5. **Functions and Structures:** Basic Functions, Different Types of Functions, Local and Global Variables, Call by Value, Call by Reference, Passing Arrays in a Function as Parameter, Recursive Function. Structures, Pointers and File Operation: Basics of Structures, Pointer Operation, Call by Reference using Pointers, Basic File Operations.
6. **Objects and Classes:** Attributes and Functions, Constructors and Destructors, Operator Overloading, Function Overloading.
7. **Inheritance and Virtual Functions:** Derived Class and Base Class, Derived Class Constructors, Overriding Member Functions, Abstract Base Class, Virtual Functions: Virtual Functions, Pure Virtual Functions, Friend Functions, Friend Class.
8. **Exception and Exception Handling:** Exception Handling Fundamentals, Exception Types.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. Robert Lafore, Object-Oriented Programming in C++.
2. Herbert Schildt, Teach Yourself C++.
3. E Balagurusamy, Object Oriented Programming with C.
4. P. J. Deitel, H. M. Deitel, C++ How to Program.
5. Joyce Farrell, Object Oriented Programming using C++.

## AMTH 208: Mathematical Statistics

4 credits

### Introduction and Specific Objectives

The course contents are designed to give students a clear idea about random variables and methods of finding the distribution of a function of random variables, Central limit theorem and Chebyshev's inequality with applications and sampling distributions. Moreover, this course is also designed to give fundamental concept of estimation theory and hypothesis testing, to obtain approximate values and confidence intervals for the unknown parameters, constructing different hypothesis testing procedures related to parametric, goodness of fit and analysis of variance tests using appropriate statistical methods and theories.

### Learning outcomes

On successful completion of this course, students will be able to: understand the basic concept of random variables, methods of finding the distribution of a function of random variables, Central limit theorem and Chebyshev's inequality with applications, Sampling distributions, basic terms of estimation theory and test of hypothesis, obtain point estimators and construct confidence intervals of parameters with applications of estimation methods and hypothesis testing.

### Course Contents

#### Part A: Sampling Distributions

1. **Population and Sample:** Concept of population, sample, parameter, statistic, random sample, probability distribution, Standard errors of statistics and their large sample approximations. Transformation of variables including square root, log, sin-inverse etc.
2. **Random Variables:** Basic concept of random variable and its types, Distribution of sum, difference, product and quotient of random variables, functions of random vectors of continuous and discrete type, Central limit theorem, other limit laws and their applications.
3. **Expectations and Generating Functions:** Conditional expectations, Chebyshev's inequality, probability generating function, characteristic function, inversion theorem.
4. **Sampling distributions:** Definition, Different sampling distributions: Chi-square ( $\chi^2$ ), Snedecor-Fisher's F and Student's t distribution, Different methods of finding sampling distribution: Analytical method, inductive method, geometrical method, method of using characteristic function, etc. Sampling from the normal distributions, Distribution of sample mean and variance and their independence for normal population, Sampling distribution of correlation and regression coefficients, frequency and their uses.

#### Part B: Inference

5. **Basics of Estimations:** Methods of estimation and criteria of estimations. Preliminaries of tests: Hypothesis, Types of hypotheses, concept of test of significance, procedures of a test, errors in testing of hypothesis, level of significance, one tailed and two-tailed tests, p-value. Tests based on different statistic.
6. **Tests:** Testing the significance of a single mean, single variance, single proportion, difference of two means and proportions, ratio of two variances and their confidence intervals. Tests and confidence intervals concerning simple correlation coefficient and regression coefficient for single and double sample. Paired t-test.

7. **Attributes and Contingency Tables:** Association of attributes, Association & disassociation, Measure of association, Attribute, contingency tables, General test of independence in an  $r \times c$  contingency table. Fisher's exact test for a  $2 \times 2$  contingency table.
8. **Goodness of fit:** Test of goodness of fit, Analysis of Variance (ANOVA): One-way, two-way classification etc.

**Evaluation:** Incourse Assessment and Attendance 30 marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, four from each part, of which **five** are to be answered, taking at least **two** questions from each part.

#### References

1. Hoog, R.V.& Craig, A.T., An introduction to mathematical statistics.
2. Prem S. Mann, Introductory Statistics, 8th Edition John Wiley & Sons.
3. Gupta, S.C. and Kapoor, V.K., Fundamental of Mathematical Statistics.
4. Richard A. Johnson and Gouri K. Bhattacharyya, Statistics: Principles and Methods.
5. John E. Freund, Miller and Miller, Mathematical Statistics with Applications.

#### AMTH 250: Math Lab II (Fortran)

**3 credits**

Problem solving in concurrent courses (e.g. Calculus, Linear Algebra, Differential Equations, Numerical Analysis and Discrete Mathematics) using FORTRAN Programming.

**Lab Assignments:** There are at least 07 assignments.

**Evaluation:** Internal assessment (Laboratory works) 40 Marks. Final examination (Lab: 3 hours) 60 Marks.

#### AMTH 299: Viva Voce

**2 credits**

Viva voce on courses taught in the Second Year.

**Introduction and Specific Objectives**

This course is designed to introduce basic notions and methods of function of a complex variable, analytic functions, complex integrations and residue calculus including branch line integrals. Use basic theorems on complex sequences and series, with a particular emphasis on power series. Calculate coefficients and radii of convergence of power series using these theorems. Demonstrate a familiarity with the basic properties of analytic functions. Apply these theorems to simple examples. State correctly the theorems of Cauchy and Morera. Calculate, using Cauchy's theorem and its corollaries, the values of contour integrals.

**Learning Outcomes**

Give an account of the concepts of analytic function and harmonic function and to explain the role of the Cauchy-Riemann equations; explain the concept of conformal mapping, describe its relation to analytic functions, and know the mapping properties of the elementary functions; Give an account of and use the Cauchy integral theorem, the Cauchy integral formula and some of their consequences; Analyze simple sequences and series of functions with respect to uniform convergence, describe the convergence properties of a power series, and determine the Taylor series or the Laurent series of an analytic function in a given region; Basic properties of singularities of analytic functions and be able to determine the order of zeros and poles, to compute residues and to evaluate integrals using residue techniques; Formulate important results and theorems covered by the course and describe the main features of their proofs; finally use the theory, methods and techniques of the course to solve mathematical problems.

**Course Contents**

1. **Complex Numbers:** The Complex Number System, Fundamental Operations with Complex Numbers, Graphical Representation of Complex Numbers, Polar Form of Complex Numbers, DeMoivre's Theorem, Roots of Complex Numbers, Equations, The  $n$ th Roots of Unity, Vector Interpretation of Complex Numbers.
2. **Complex Function and its Derivative:** Functions, Limits and Continuity, The Complex Derivative, The Derivative and Analyticity, Cauchy-Riemann Equation, Harmonic Functions, Some Physical Applications of Harmonic Functions.
3. **Integration in the complex plane:** Definite Integrals, Contour Integrals, Antiderivatives, Cauchy-Goursat Theorem, Cauchy Integral Formula, Liouville's Theorem, Fundamental Theorem of Algebra, Maximum Modulus Principle.
4. **Series:** Taylor's and Laurent's expansion, singularity, Poles and Residues, Cauchy's Residue Theorem, Residue at Infinity, Zeros of Analytic Functions.
5. **Conformal mappings:** Elementary conformal mappings and their geometric properties. The bilinear transformations.
6. **Complex Beta and Gamma functions:** Gauss's of Gamma function, Gauss's multiplication theorem of Gamma function, The Weierstrass form of the Gamma function, Hankel form of the Gamma function, Weierstrass Form equivalent to Euler Form, limiting relation between Gamma and Beta function, Euler's Gamma function, logarithmic derivative of gamma function, Stirling's formula.
7. **Fourier Series:** Fourier series and its convergence. Fourier sine and cosine series. Properties of Fourier series. Operations on Fourier series. Complex form. Applications of Fourier series.
8. **Fourier transforms:** Fourier transforms. Inversion theorem. Sine and cosine transforms. Transform of derivatives. Transforms of rational function. Convolution theorem. Parseval's theorem.

**Evaluation:** In-course Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. J. H. Mathews and R. W. Howell, Complex analysis for mathematics and Engineering.
2. E. B. Saff and A. D. Snider, Fundamentals of complex analysis with applications in Engineering and Science.
3. D. G. Zill and P. D. Shanahan, A first course in complex analysis with applications.
4. A. D. Wunsch, Complex variables with applications.

## AMTH 302: Theory of Numbers and Groups

3 credits

### Introduction and Specific Objectives

According to Karl Friedrich Gauss, "Mathematics is the queen of sciences and the theory of numbers is the queen of mathematics." Number theory is arguably one of the oldest and most fascinating branches of mathematics. This fascination stems from the fact that there are a great many theorems concerning the integers, which are extremely simple to state, but turn out to be rather hard to prove. The objective of this course is to introduce continued fraction of rational and irrational numbers, Linear Diophantine equation, Quadratic Residues, Congruence with its application in different fields like Scheduling Round-Robin tournament, checking ISBN as well as Cryptography. In another part of this course, where the notion and classification of the groups are in concern.

### Learning Outcomes

- Ability to describe and use the continued fraction algorithm to find representations of rational and quadratic irrational numbers.
- Familiar with linear Diophantine equation and Linear Congruencies, Chinese remainder theorem.
- Get proper knowledge of congruence and can apply in different real life problems.
- A sound knowledge of groups and subgroups, symmetric and cyclic groups, normal subgroups and quotient groups, groups of small orders.

### Course Contents

1. **Continued fractions:** Simple continued fraction, Convergent of continued fraction, Infinite Continued fraction- Periodic and Non-Periodic.
2. **Linear Diophantine equations and Congruence:** Linear Congruence, Solution of System of Linear Congruencies with single variable but different moduli and different variables but single modulus, Chinese remainder theorem.
3. **Application of congruence:** Divisibility test, Round Robin tournament schedule, ISBN Check Digits, Pseudorandom Generators etc.
4. **Arithmetic of quadratic fields:** Quadratic Integers, Quadratic Congruence, Quadratic Residue and Euclidean quadratic Fields, Representation by sum of two and four squares only statements (No Proof).
5. **Application of Number theory in Cryptography:** Encryption Schemes, Digital Signatures, Fault-Tolerant Protocols and Zero-Knowledge Proofs, RSA encryption method.
6. **Notions of Group:** Definition and examples of groups and subgroups, symmetric and cyclic groups, normal subgroups and quotient groups, groups of small orders.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. Kenneth H. Rosen, Elementary Number Theory.
2. S. G. Telang, Number Theory.
3. G.H. Hardy & E.M. Wright, An Introduction to Theory of Number.
4. Oded Goldreich, Foundations of Cryptography.
5. WK Nicholson, Introduction to Abstract Algebra.

**Introduction and Specific Objectives**

The first part of this course equips students with the fundamental tools required in order to solve simple partial differential equations (PDEs). This includes an understanding of how to classify PDEs and what this classification means physically. The method of characteristics is then introduced in order to solve First order quasi-linear PDEs. Then the course focuses on solving second order PDEs (mainly the heat equation, the wave equation, and Laplace's equation), first analytically by employing separation of variables. The second part of the course focuses on the study of Integral Equations and the relationship between the integral equations and ordinary differential equations and how solved the linear and non-linear integral equations by different methods with some problems which give rise to Integral Equations.

**Learning Outcomes**

The students will be introduced to partial differential equations (PDEs) and some basic classification of PDEs. At the end of the course the students will be able to solve first order partial differential equations using the method of characteristics. The students will be able to recognize classical PDEs describing physical processes such as diffusion, wave propagation, etc. and will learn to classify second-order PDEs as elliptic, hyperbolic or parabolic. They will learn to solve analytically the heat and wave equations (in one space variable) using the method of separation of variables and eigenfunction expansion method. The students will also learn to solve Laplace's equation (in two space variables) on rectangular and circular domains and also get a basic idea of maximum-minimum principle. The students will be introduced to various types of integral equations and their relations with initial value problems and boundary value problems. They will know how to solve the linear and nonlinear integral equations using various methods.

**Course Contents****Part A: Partial Differential Equations**

1. **Mathematical formulation** and modeling of physical systems in PDE, well-posed problems, usual operators and classes of equations, boundary conditions, IVP, BVP, EVP, IBVP.
2. **First order equations:** Methods for finding general solutions, constant-coefficient advection equation, linear and quasi-linear equations, methods of characteristics, IVP for conservation laws and applications. Shocks and expansion fans. Charpit's method, Solution by ODE method and separation variables.
3. **Second order PDE:** General equations and classifications, canonical forms, constant coefficient equations. Methods of solutions: separation of variables and eigenfunction expansion methods for one dimensional heat (heat flow in a rod) and wave equations, d'Alembert's formula, nonhomogeneous problems. Initial BVPs. Two dimensional heat and wave equations.
4. **The potential equation:** Method of separation variables for Laplace and Poisson equations, Dirichlet and Neumann problems in rectangular, circular (disk), partially bounded and unbounded domains, Properties of Harmonic functions. Maximum-Minimum principles, Mixed BVPs, Eigenvalue problem, Helmholtz equation, Nonhomogeneous boundary conditions.

**Part B: Integral Equations**

5. **Integral equations:** Classification of integral equations: Volterra and Fredholm integral equations, Singular and Integro-differential equations. Converting Volterra equation to ODE and IVP to Volterra integral. Converting IVP and BVP to Fredholm integral equations. Volterra equation of the first and second kind, various types of Fredholm integral equations.
6. **Methods of solutions of integral equations:** Successive approximations and substitutions, Adomian decomposition methods for Volterra and Fredholm integral equations. Nonlinear integral equations: Picard's method, Adomian decomposition method.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.



**Three** out of **Five** should be answered from **Part A** and **Two** out of **Three** should be answered from **Part B**.

### References

1. Larry C. Andrews, Elementary Partial Differential Equations with Boundary Value Problems, Academic Press College.
2. Paul DuChateau and David Zachmann , Applied Partial Differential Equations.
3. Richard Haberman, Elementary Partial Differential Equations with Fourier series and BVPs, Prentice-Hall International Editions.
4. Yehuda Pinchover and Jacob Rubinstein, An Introduction to Partial Differential Equations, (Cambridge University Press).
5. M. Rahman, Integral Equations and their Applications (WIT press).

### AMTH 304: Mathematical Methods

**4 credits**

#### Introduction and Specific Objectives

This is the level 3 course in the Department of Applied Mathematics and designed to solve various Mathematical problems using analytical methods. Main objective of this course is to establish a relationship between mathematics and application to other disciplines such as physical science and engineering students. This course also aims:

- to introduce power series solution method about ordinary and singular points to solve ordinary and partial differential equations with constant and/or variable coefficients arises in various fields of physical and engineering areas,
- to introduce the concept of Eigen function method and Sturm-Liouville problem arises in various fields,
- to introduce Special types of differential equations in physical and engineering fields used by Physicists and engineers from which special functions such as Bessel's function, Legendre function, Hermite function, Laguerre function, hypergeometric and confluent hypergeometric functions with their applications,
- to introduce Green's function to solve steady state heat equation, wave equation, Laplace equation in 2D and 3D etc.
- to introduce the Students to understand Laplace transforms and inverse Laplace transform to solve linear/nonlinear problems of Mathematical models of different physical/ engineering areas with constant coefficients and/or variable coefficients.

#### Learning Outcomes

This course is aligned with the following Applied Mathematics program learning outcomes:

- Students will be able to solve mathematical problems using analytical methods and recognize the relationships between different areas of mathematics and the connections between mathematics and application to other disciplines.
- Students will be able to apply series solution method about ordinary and singular points to solve various physical and engineering problems arises as models in terms of ordinary and partial differential equations.
- Students will understand Eigen function methods and Sturm-Liouville problems and will be able to solve such problems arises in various fields.
- Students will be able to understand and use Green's function to solve ordinary and partial differential equations such as steady state heat equation, wave equation, Laplace equation in 2D and 3D etc.
- Students will understand special functions such as Bessel's function, Legendre function, Hermite function, Laguerre function, hypergeometric and confluent hypergeometric functions with their applications.
- Students will understand the most effective transformation to transform physical space to Laplace space and the inversion to solve linear/nonlinear problems of Mathematical models of different physical/ engineering areas with constant coefficients and/or variable coefficients.

## Course Contents

1. **Series solution of differential equations:** Series solution about ordinary and singular point, regular and irregular singular point of a linear ODE, distinct roots not differing by an integer, repeated root of an indicial equation, distinct roots differing by an integer, Frobenius method for 2<sup>nd</sup> order ODE, Derivative method.
2. **Eigenfunction methods and Sturm-Liouville Theory:** Adjoint, eigenfunction properties, regular Sturm-Liouville boundary value problems. Nonhomogeneous boundary value problems. Singular Sturm-Liouville boundary value problems. Oscillation and comparison theory.
3. **Green's function and Fredholm alternative:** Solution by eigenfunction expansion, Inverse of differential operator, Green's function via Delta function, General linear boundary value problem, General Green's Function, Applications to steady state heat equation and wave equations in 1D, steady state heat equation and potential flow problems (Laplace) in 2D and 3D, Fredholm alternative.
4. **Special functions:** Bessel functions (differential equations; series solutions; integral representations), Bessel functions of 1<sup>st</sup> and 2<sup>nd</sup> kind, applications; Legendre equations and Legendre functions and their properties, generalization, applications; orthogonal polynomials: Legendre/Jacoby; Hermite; Laguerre, Chebyshev, Hypergeometric, confluent Hypergeometric and their applications.
5. **Laplace transform and Inverse Laplace Transform:** Definition, Laplace transform of some elementary functions; sufficient conditions for the existence of Laplace transform; some important properties of Laplace transform: translations, derivatives of a transform, transforms of integrals; initial and final value theorem; Laplace transforms of some special functions (periodic functions, Dirac Delta functions). Inverse Laplace, some important properties of the inverse Laplace transform; partial function decompositions; convolution theorem; Heaviside's expansion formula; evaluation of integrals.
6. **Applications of Laplace transform:** Solving differential equations (ordinary and partial) using the Laplace transform, solving differential equations involving unit step functions and the Dirac Delta function, solving systems of ODEs using Laplace Transforms. Use of Laplace Transform techniques to model application problems from the physical sciences.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. I. Stakgold, MJ Holst, Green's Functions and Boundary Value Problems.
2. Erwin Kreyszig, Advanced Engineering Mathematics.
3. N. N. Lebedev- Special functions and their applications.
4. M. R. Spiegel - Laplace Transform.
5. KT Tang – Mathematical methods for Engineers and Scientists.

## AMTH 305: Numerical Methods II

**3 credits**

### Introduction and Specific Objectives

This course is a study of mathematical techniques used to model physical phenomena arising from different branches of science and engineering. It involves the development of mathematical models and the application of the computer to solve engineering problems using the following computational techniques: curve fitting, Spline Interpolation, solution methods for nonlinear system of equations, numerical solution of differential equations, ordinary Differential Equations, with applications to engineering problems.

The objective of this course is to provide students with the skills, knowledge and attitudes required to determine approximate numerical solutions to mathematical problems which cannot always be solved by conventional analytical techniques, and to demonstrate the importance of selecting the right numerical technique for a particular application, and carefully analyzing and interpreting the results obtained.

## Learning Outcomes

At the conclusion of the course the student will be able to

- use least squares approximation to find the best fit curve for a given set of data points
- choose an appropriate numerical solution method based on the properties of the given non-linear system
- find numerical solution of a initial value problems (IVP) by different single and multistep methods
- construct numerical methods for the numerical solution of boundary-value problems
- accuracy properties of these methods.

## Course Contents

1. **Curve Fitting and Approximation:** Spline Interpolation and Cubic Splines, Least Squares Approximation.
2. **Approximating Eigenvalues:** Eigenvalues and eigenvectors, the power method, Convergence of Power method, Inverse Power method, Rayleigh Quotient Method, Householder's method, Q-R method.
3. **Nonlinear System of Equations:** Fixed point for functions of several variables, Newton's method, Quasi-Newton's method, Conjugate Gradient Method, Steepest Descent techniques.
4. **Initial Value Problems for ODE (Single-step methods):** Euler's and modified Euler's method, Higher order Taylor's method, Runge-Kutta methods.
5. **Multi-step Methods:** Adams-Bashforth, Adams-Moulton, Predictor-Corrector and Hybrid methods, variable step-size multi-step methods, error and stability analysis.
6. **Higher Order Methods:** Methods solving higher order differential equations and systems of differential equations.
7. **Boundary Value Problem for ODE:** Shooting method for linear and nonlinear problems, Finite difference methods for linear and nonlinear problems.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. R.L. Burden and J.D. Faires – Numerical Analysis.
2. K. Atkinson – Introduction to Numerical Analysis.
3. M.A.Celia and W.G. Gray – Numerical Methods for Differential Equations.
4. L.W. Johson & R.D. Riess –Numerical Analysis.
5. John H. Mathews –Numerical Methods for Mathematics, Science and Engineering.

## AMTH 306: Mechanics

**3 credits**

### Introduction and Specific Objectives

Learning the engineering approach to problem solving is one of the more valuable lessons to be learned in an introductory dynamics course. Mechanics is a base of various branches of modern physics, applied mathematics and engineering. A thorough understanding of mechanics serves as a foundation of studying different areas in sciences. Specifically, this course is concentrated with the behavior of bodies under the action of forces.

## Learning Outcomes

On successful completion of this course students will be able to

- to solve problems of body in motion using Newton's law
- to understand dynamics of particles in rectangular coordinates
- to understand and apply equation of planetary motion
- to understand moments and products of inertia
- to understand kinematics of rigid bodies including rotational motion and angular motion relative to fixed and rotating reference frame.

## Course Contents

1. **Newtonian Mechanics:** Newton's law of motion, Inertial frames and the law of inertia, Law of multiple interactions, Center of mass.
2. **Dynamics of a particle (Rectangular coordinates):** Kinematics, kinetics: force-mass acceleration method, Dynamics of rectilinear motion, curvilinear motion.
3. **Planetary motion:** Equation of motion under a central force, differential equation for the orbit, Orbits under an inverse square law.
4. **Vibrations:** Free vibrations of particles, forced vibrations of particles, rigid body vibrations.
5. **Mass Moments and Product of Inertia:** Moments of inertia of thin plates, Mass moment of inertia by integration, Mass product of inertia; Parallel axis theorem, Products of inertia by integration; thin plates, Principal moments and principal axes of inertia.
6. **Planar kinematics of Rigid Bodies:** Plane angular motion, rotation about a fixed axis, relative motion of two points in a rigid body, motion relative to a rotating reference frame.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. David Acheson, From Calculus to Chaos An Introduction to Dynamics.
2. Mary Lun, A first course in Mechanics.
3. R. Douglas Gregory, Classical Mechanics.
4. Andrew Paytel Engineering Mechanics: Dynamics.

## AMTH 307: Hydrodynamics

**3 credits**

### Introduction and Specific Objectives

This course is designed for students who have studied some basic courses in physics and are familiar with the basics of thermodynamics and vector calculus. This course covers the development of the fundamental equations of fluid dynamics and their simplifications for several areas of hydrodynamics and the application of these principles to the solution of realistic problems. Topics include the principles of conservation of mass, momentum, inviscid flows, potential flows, complex potential of source, sink, doublet and vortex, Joukowski transformation, flow past circular cylinder, open-channel water flows, surface waves, including wave velocities, propagation phenomena, and description of finite amplitude waves in shallow water. The objectives of this course are to give a general overview to students about use of Hydrodynamics in solving realistic problems mathematically. A specific objective is to illustrate examples from everyday experience so that the student can develop an intuitive understanding which can then be applied in other contexts.

### Learning Outcomes

After successful completion of this course, the students will be able to understand the basic properties of water, different types of flow and the standard computational flow visualization techniques. They will know the Bernoulli, continuity and Euler equations, its applications in various fields and the limitations (if any) of using these equations. The learners will be able to study the potential flow, its solution techniques, complex potential and its uses in solving Uniform Flow, Source, Sink, Vortex and Doublet problems. The students will study the Joukowski transformation and its applications in solving complex flow problems and will be able to make the superposition of several standard flow problems mentioned above in solving more complex problems. Besides they will be able to understand the characteristics of open channel flow i.e., flows in River, Canal etc. which has many obvious applications in Bangladesh. Finally, the students will study the surface waves that generally exist in open channel flows, and Finite Amplitude Waves in Shallow Water.

## Course Contents

1. **Water Flows:** Water and its properties; Steady and Unsteady flows; Uniform and Non-uniform flows; Rotational and Irrotational flows; Compressible and Incompressible flows; Flow visualization: Streamlines; Streaklines; and Pathlines.
2. **Bernoulli's Equation:** Hydrostatics; Liquid flow under conservative force; Bernoulli's theorem; Applications of Bernoulli's equation: Torricelli's theorem; Sluice gate and its applications; Discharging a tank; etc.
3. **Equations of Motion:** Concept of control volume; Equations of continuity; Viscous and Inviscid flows; Euler's equation of motion; The hydrodynamic equations; The shallow-water equations. Vorticity; Helmholtz's vorticity equation; Circulation.
4. **Potential Flow:** Potential flow; Stream function and Velocity potential in Cartesian and Polar-coordinates; Relation between stream function and velocity potential; Three-dimensional potential flows: Velocity potential; Stoke's stream function.
5. **Complex Potential:** Complex potential and complex velocity; Stagnation points; Uniform flows; Source; Sink; Vortex and Doublet. Complex potential due to source; sink; vortex and doublet.
6. **Transformations:** Joukowski's transformation: Transformation of circle into straight line and ellipse; Method of images; Circle's theorem; Flow past a circular cylinder with circulation and without circulation; Pressure distribution and Pressure coefficient on the surface of the Cylinder.
7. **Open-Channel Water Flows:** General characteristics of Open-Channel Flow. Effects of Reynolds number, Weber number and Froude number on Open Channel Flow, Uniform flow approximations; The Chezy and Manning Equations, Varies Flows; The Hydraulic jump.
8. **Waves in Water:** Surface waves; Small amplitude plane waves; Propagation of surface waves; Complex potential for travelling waves. Sound waves; Finite amplitude waves in shallow water.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. H. R. Vallentine, Applied Hydrodynamics.
2. Bruce R. Munson, Donald F. Young, Theodore H. Okiishi, Fundamentals of Fluid Mechanics.
3. L. M. Milne Thomson, Theoretical Hydrodynamics.
4. Yunus Cengel & John Cimbala, Fluid Mechanics: Fundamentals and Applications.

## AMTH 308: Introduction to Financial Mathematics

**3 credits**

### Introduction and Specific Objectives

This course is a basic introduction to finance. It starts by making an introduction to the value of money, interest rates and financial contracts, in particular, what are fair prices for contracts and why no-one uses fair prices in real life. Then, there is a review of probability theory followed by an introduction to financial markets in discrete time. In discrete time, one will see how the ideas of fair pricing apply to price contracts commonly found in stock exchanges. The next block focuses on continuous time finance and contains an introduction to the basic ideas of Stochastic calculus. There is an overview of Actuarial Finance also. This course is a great introduction to finance theory and its purpose is to give students a broad perspective on the topic." The course unit aims to enable students to acquire active knowledge and understanding of some basic concepts in financial mathematics including stochastic models for stocks and pricing of contingent claims.

### Learning Outcomes

On successful completion of the course, students will be able to

- gain knowledge of basic financial concepts and financial derivative instruments
- use the fundamentals of no-Arbitrage pricing concept
- apply basic probability theory to option pricing in discrete time in the context of simple financial models
- understand fundamental knowledge of Stochastic analysis (Ito Formula and Ito Integration)
- understand Black-Scholes formula and get the concept of Introduction to actuarial mathematics
- learn price financial derivatives such as options.

## Course Contents

1. **Introduction to Finance:** Definition of finance, types of finance, major financial decisions, goals of finance, functions of financial institutions and financial Market, difference between the capital markets and the money markets.
2. **Time Value of Money:** Definition and concepts-cash flow, discounting and compounding, present value, future value, annuities, mixed streams, effective annual interest rate, amortization.
3. **Interest rates, Bond and Stock Valuation:** Interest rates and required returns, Term structure of interest rates, important bond features, different types of bond, valuation fundamentals and bond valuation; Difference between debt and equity, features of both common and preferred stock, basic stock valuation using zero-growth, constant-growth, and variable growth models.
4. **Overview of basic concepts in securities markets:** Exchange-traded markets; Over-the-counter markets; Forward contracts; Future contracts; Options; Types of traders, etc.
5. **Stochastic models for stock prices:** Continuous-time stochastic processes; Wiener processes; The process for a stock price; The parameters; Itô's lemma; The lognormal property of stock prices.
6. **Hedging strategies and managing market risk using derivatives:** Financial derivatives; European call and put options; Payoff diagrams, short selling and profits; Trading strategies: Straddle, Bull Spread, etc; Bond and risk-free interest rate; No arbitrage principle; Put-call parity; Upper and lower bounds on call options.
7. **Binomial option pricing model:** One-step binomial model and a no-arbitrage argument; Risk-neutral valuation; Two-steps binomial trees; Binomial model for stock price; Option pricing on binomial tree; Matching volatility  $\sigma$  with  $u$  and  $d$ ; American put option pricing on binomial tree.
8. **Risk-neutral Portfolio:** Risk-neutral valuation, replication and pricing of contingent claims.
9. **Black-Scholes analysis:** Black-Scholes model; Black-Scholes Equation; Boundary conditions for call and put options; Exact solution to Black-Scholes equation; Delta-hedging; the Greek letters; Black-Scholes equation and replicating portfolio; Static and dynamic risk-free portfolio; Option on dividend-paying stock; American put option.

**Evaluation:** In-course Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. L J Gitman, Principles of Managerial Finance 12th Edition.
2. J. Hull, Options, Futures and Other Derivatives, 8th Edition, Prentice-Hall, 2012.
3. P. Wilmott, S. Howison and J. Dewynne, The Mathematics of Financial Derivatives: A Student Introduction, Cambridge University Press, 1995.

## AMTH 309: Optimization Techniques

**4 credits**

### Introduction and Specific Objectives

This course is an introductory and practical course to the study of operations research application in mining projects. It is designed primarily for mining engineering students to replicate what is happening in the mining industry in classroom so as to be able to apply the knowledge and skills gained during and after course of study to real life situations they might face in the industry. It involves demonstration of principles and techniques of operations research using real life projects. Topics to be covered include operation research and model formulation, solution of the operation research model, phases of an operation research study, techniques of operation research or operations research solution tools such as Linear Programming (LP) (Two phase (two variables) LP, Three phase (three variables) LP), Transportation models, Nonlinear Programming etc.

The objectives of this course are to introduce students to the techniques of operations research in mining operations, provide students with basic skills and knowledge of operations research and its application in mineral industry and introduce students to practical application of operations research in big mining projects.

### Learning Outcomes

Students that successfully complete this course will be able to

- have a basic idea of Linear programming Problems ( LPP ), Formulate the LPP, Conceptualize the feasible region, Solve the LPP with two variables using graphical method
- know about Simplex Method and also different types of simplex Method, Solve the LPP using different types of simplex method
- know about Dual Simplex Method, , Formulate the dual problem from primal and its solution.
- analyze the sensitivity of decision variable(s)
- recognize and formulate transportation, assignment problems and drive their optimal solution
- find Nonlinear Programming Problems solution using different types of method.

### Course Contents

1. **Introduction to Linear Programming:** Basic definitions, Formulation of linear programming problems, Graphical solutions.
2. **Simplex Method :** Simplex method, Setting up the Simplex Method, The Algebra of the Simplex Method, The Simplex Method in Tabular Form, Solution and Convergence: Two phase method, Big-M simplex method.
3. **Duality Theory:** The Essence of Duality Theory and Primal-Dual Relationships, Economic Interpretation of Duality, Duality of linear programming and related theorems (No Proof), Dual simplex method.
4. **Sensitivity Analysis:** Analysis of the effect of changing various parameters in linear programming problems such as right hand side of the constraints, cost coefficients, addition of a new constraint, deletion of a constraint etc.
5. **Transportation and Assignment Problem:** Introduction, Formulation, Solution procedure, applications.
6. **Nonlinear Programming:** Introduction, Unconstrained problem, Lagrange Method for Equality constraint problem, Kuhn-Tucker Method for Inequality constraint problem, Quadratic programming problem.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. The Wayne L Winston, Operations Research: Applications and Algorithms, Indiana University.
2. Sharma J K, Operations Research: Theory and Applications, (2013), Macmillan Pub India.
3. Taha H A, Operations Research: An Introduction, ( 2009),Prentice-Hall of India.
4. Hillier, F.S. and G.J. Lieberman, 'Introduction to Operations Research', 9th Ed., 2010, McGraw Hill, New York.

### AMTH 350: MATH LAB III (MATLAB)

**3 credits**

Problem solving in concurrent courses on First year to Third year using MATLAB Programming.

**Lab Assignments:** There are at least 07 assignments.

**Evaluation:** Internal assessment (Laboratory works) 40 Marks. Final examination (Lab, 3 hours) 60 Marks.

### AMTH 399: Viva Voce

**2 credits**

Viva voce on courses taught in Third Year

**Introduction and Specific Objectives**

This course covers the extended course on mathematical analysis (AMTH 201) which is introduced as one of the compulsory courses for fourth year undergraduate level of applied mathematics. It is a combined course on topology and functional analysis. The topology part of this course is to provide for the students an introduction to theory of metric and topological spaces with emphasis on those topics that are important to higher mathematics. This part also focuses on the basic notions of metric and topological spaces, properties of continuous mappings selected types of topological spaces (compact and connected spaces) and basic theorems on topological spaces.

Functional analysis forms the basis of the theory of operators acting in infinite dimensional spaces. It has found broad applicability in diverse areas of mathematics (for example, spectral theory). Students will be introduced to the theory of Banach and Hilbert spaces. This will be followed by an exposition of four fundamental theorems relating to Banach spaces (Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem).

**Learning Outcomes**

By the end of this course, students will understand basic concepts and results in topology and functional analysis, be able to solve routine problems and have developed skills in applying the techniques of the course to unseen situations. Students also will be able to appreciate how ideas from different areas of mathematics combine to produce new tools that are more powerful than would otherwise be possible, and will understand how topology and functional analysis underpin modern analysis. This provides the basic tools for the development of such areas as quantum mechanics, harmonic analysis and stochastic calculus. It also has a very close relation to measure and integration theory.

**Course Contents**

1. **Topological Spaces:** Definition and examples (discrete, indiscrete, cofinite, cocountable topologies), closed and open set, interior, exterior and boundary points, derived set, cluster point of a set, dense set, relative topology, neighborhood system. Continuity.
2. **Separation axioms:** Properties of Hausdorff spaces. Product spaces. Countability of topological spaces.
3. **Properties of metric spaces:** Complete and incomplete metric spaces, Baire's category theorem. Necessary and sufficient condition for compactness. Heine-Borel theorem. Finite intersection property. Equivalence of sequential compactness, Bolzano-Weierstrass property, Totally boundedness, Lebesgue number and compactness in a metric spaces. Cantor set.
4. **Connectedness in metric spaces:** totally disconnected spaces, components of space, locally and path wise connected spaces.
5. **Normed Linear Spaces:** Definitions and examples, Cauchy-Schwarz inequality, Parallelogram law, Metric derived from a norm. Holder and Minkowski inequalities for finite and infinite sums, and integrals.  $l^p$  space in a metric space, Norm of  $L_p$ ,  $l^p$  and Sobolev spaces, Banach spaces. Riez's lemma.
6. **Linear operators:** Boundedness and continuity, Linear operators in finite dimensional spaces. Spaces of bounded linear operators. Open mapping theorem, Closed graph theorem, and their applications, Uniform boundedness principle.
7. **Inner product Space:** Inner product space and Hilbert Space, polarization identity, orthogonal and orthonormal sets in Hilbert space, Bessel's inequality.
8. **Fixed point theorems:** Contraction mapping, Banach fixed point theorem, Applications of fixed point theorems.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.



## References

1. G.F. Simmons, Introduction to Topology and Modern Analysis.
2. S. Lipschutz, General Topology.
3. Fatema Chowdhury and Munibur Rahman Chowdhury, Essentials of Topology and Functional Analysis.
4. E. Kreyszig, Introduction to Functional Analysis with Applications.
5. J N Reddy, Applied Functional Analysis and Variational Methods in Engineering.

## AMTH 402: Fluid Dynamics

3 credits

### Introduction and Specific Objectives

Fluid flows are the most common phenomena in nature and life in the planet, Earth. The knowledge of fluid dynamics is very much required to solve scientific and technological problems including atmospheric and oceanic circulation, aerodynamics, blood flow, etc. Considerable progress has been made in the mathematical modelling of fluid flows and this has greatly improved our understanding but there is still much to discover. This course introduces students to the mathematical description of fluid flows and the solution of some ideal flow problems.

### Learning Outcomes

On successful completion of this course students will be able to

- to understand continuum fluid and Newton's law of viscosity
- to derive Navier-Stokes equation and learn its significance
- to solve some ideal flow problems analytically
- to understand the characteristics of very slow fluid motion
- to understand concepts of boundary layer and apply in solving some common/real problems
- to understand thermal boundary layer and apply in solving ideal flat plate problems.

### Course Contents

1. **Fundamental concepts:** Fluid as a continuum, Newton's law of viscosity, Newtonian and non-Newtonian fluids, Body and surface forces, Stress and Rate of strain and their relation.
2. **Navier-Stokes equation:** Navier-Stokes equations in different coordinate systems, Vorticity Transport Equation, Nondimensionalization, Dimensionless parameters, Reynolds similarity.
3. **Unidirectional Flow, Exact solutions of the Navier-Stokes equations:** Couette flows, plane Poiseuille flow, Flow through a circular pipe, the Hagen-Poiseuille flow, Flow between two coaxial cylinders and concentric circular cylinders, Pulsating flow between parallel surfaces, Stoke's first and second problems.
4. **Very Viscous Flow:** Introduction, Low Reynolds number flow past a sphere, Swimming at low Reynolds number, Uniqueness and reversibility of slow flows, Flow in a thin film, Lubrication theory.
5. **Boundary layers:** General concepts and properties of boundary layer. Prandtl's boundary layer equations, boundary layer Separation, Similar and nonsimilar solutions of the boundary layer equations, Flow in a convergent channel, Flow past a wedge, Boundary layer on a flat plate, Boundary layer flow with pressure gradient, Karman's integral equation, Karman-Pohlhausen method.
6. **Thermal boundary layer:** Energy equation, Thermal boundary layer simplifications, Natural and Forced flows, Parallel forced flow past a flat plate at zero incidence, Natural flow past a horizontal/vertical plates.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. D.J. Acheson, Fluid Dynamics.
2. I.G. Currie, Fundamental Mechanics of Fluids.

3. Frank M. White, Viscous Fluid Flow.
4. H. Schlichting, Boundary Layer Theory.

### AMTH 403: Physical Meteorology

3 credits

#### Introduction and Specific Objectives

- To produce graduates who possess knowledge of physical structure, properties, behavior and physical processes of the atmosphere.
- To produce graduates who have a general knowledge of precipitation pattern, formation of cloud and cloud classification, high and low pressure, temperature and their impacts on environment.
- To produce graduates who will be able to understand the behavior of the atmospheric high and low pressure regions from which various atmospheric phenomena (e.g. tornadoes, cyclone, thunderstorm, cold wave etc.) produced and spread.
- To produce graduates who have a general knowledge of a range of atmospheric phenomena and applications, and have expertise in one or more program sub-disciplines or related interdisciplinary areas.
- To produce graduates who are equipped to contribute to solving problems in the atmospheric sciences and related disciplines.

#### Learning Outcomes

- Students can understand various layers of atmosphere in terms of height and temperature
- and the properties of individual layers to realize the physical phenomena at the layers
- Students can demonstrate knowledge of the typical vertical variation of the basic variables used to quantify the atmospheric state, including temperature, pressure, humidity, winds, and natural and anthropogenic particles
- Students can demonstrate knowledge of climate and climate change, together with the possible influences that humans have on diverse climate phenomena
- Students can demonstrate knowledge of the forces that drive three-dimensional atmospheric motions
- Students can demonstrate knowledge of clouds and their formation mechanisms, together with the precipitation types and other materials that precipitation cleanses from the air
- Students can demonstrate the physical processes of formation of thunderstorm, various stages and safety of thunderstorm
- Students can demonstrate knowledge of a variety of mesoscale and small-scale atmospheric phenomena, including tropical storms, severe thunderstorms, and tornadoes

#### Course Contents

1. **Meteorological Concepts:** Meteorology, synoptic meteorology, Climatology, Physical meteorology, Dynamic meteorology, Agricultural meteorology, applied meteorology.
2. **Atmosphere:** Origin of the atmosphere, layering of the atmosphere; troposphere, stratosphere, mesosphere, thermosphere, exosphere and other layers of atmosphere, Composition of the atmosphere, homogeneous atmosphere, height of homogeneous atmosphere.
3. **Thermodynamics of dry air:** Pressure, temperature and ideal gas law; The Maxwell-Boltzmann distribution, hydrostatic equilibrium, surface pressure and mass of the atmosphere, surface pressure and sea level pressure, Heating, working and the First law; Enthalpy and the second law.
4. **Thermodynamics of moist air:** Six ways of quantify moisture content, potential pressure, potential temperature, static stability of moisture non-condensing air. The Clausius-Clapeyron equation, level of cloud formation.
5. **Atmospheric Radiation:** Solar radiation, Characteristic of Sun, nature of solar radiation, Geographical and seasonal distribution of solar radiation, Deposition of solar radiation with and without cloudy skies. Solar radiation and Earth-troposphere system, terrestrial radiation, characteristics and transmission through the atmosphere, Greenhouse effects, Causes of greenhouse effects, future trends of G H effects, Sea level changes, Impact of 1-meter sea level rise in Bangladesh.

6. **Cloud and cloud formation:** Cloud formation, cloud classification, various types of clouds, cloud droplet growth, droplet growth by diffusion and condensation, terminal velocity of falling drops. Lightning and thunder, formation and various stages of thunderstorm and thunderstorm safety.
7. **Vorticity:** The circulation theorem, vorticity, the vorticity equation in Cartesian and isobaric coordinates, scale analysis, potential vorticity, Ertel potential vorticity, shallow water potential vorticity conservation.
8. **Tropical Cyclone:** Formation stage, Immature stage, mature stage, terminal stage; Climatological conditions for tropical cyclone formation, North Indian Ocean, Large scale conditions associated with tropical cyclone formation.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. J. Houghton, The Physics of Atmospheres.
2. James R .Holton: An Introduction to Dynamic Meteorology
3. Rodrigo Caballero, Lecture notes in Physical Meteorology.
4. R. R. Rogers and M. K. Yau, A short course in cloud physics.
5. Grant W. Petty, A first course in Atmospheric Radiation.

## AMTH 404: Elementary Hydrology

**3 credits**

### Introduction and Specific Objectives

This course contains all the elementary topics of hydrology and focuses particularly on the mathematical analysis of various hydrologic model. It covers the basic definitions, classifications and historical developments of hydrology as well as description and derivation of some well-known hydrologic models. This leads to an investigation of various applied analysis such as evapotranspiration and precipitation analysis, rainfall and runoff relation etc. It also includes the study of  $\Phi$  index and hydrographs which have many practical applications.

The first objective of this course is to introduce the basics definitions, classifications, historical development and common terminologies of hydrology and discuss different hydrologic models. Another major goal is to explore different aspects of hydro-meteorology such as atmosphere structure, solar radiation, precipitation and climate of the Indian subcontinent. Study evaporation, transpiration and evapotranspiration and analysis of how these things contribute to precipitation is also a concerned topic. Study of hydrograph and unit hydrograph theory and their applications are also included.

### Learning Outcomes

The students will be able to determine why Hydrology is necessary by knowing about the basic definitions, historical development and scopes of Hydrology, Hydrologic Cycle and the Global water Budget. They will have some preliminary ideas about Hydrologic System model, Hydrologic model classification and Black Box model. The students will be able to determine how the atmosphere behaves and precipitation occurs by having knowledge of evaporation, transpiration, evapotranspiration. They will learn about methods of precipitation measurements and other meteorological observations and will be able to determine the effect of altitude and temperature on various forms of precipitation. They will know how infiltration and redistribution of water in the ground occur, the importance of vadose zone, soil moisture and ground water in hydrologic cycle, relation between rainfall and runoff, source of stream flow, excess rainfall and direct runoff. They will be able to solve various types of rainfall-runoff problems using infiltration equation, SCS method and  $\Phi$  index method. The students will be capable of drawing hydrograph and using it to analyze rainfall-runoff relation. They will be able to apply the unit hydrograph and the synthesized unit hydrograph theory in various real life problems.

## Course Contents

1. **Introduction:** Definition and scope of Hydrology, Hydrologic Cycle, Hydrologic System model, Hydrologic model classification, the development of Hydrologic Black Box model, Historical development, the Global water Budget.
2. **Hydro-meteorology:** Introduction, constituents of the atmosphere, vertical structure of the atmosphere, solar radiation, the general circulation formulation of precipitation, types of precipitation, forms of precipitation, evaporation, transpiration, evapotranspiration, precipitation measurement, climate and weather seasons in the Indian subcontinents., meteorological observations.
3. **Topography:** Watershed delineation, topographic effect (altitude, temperature) on precipitation.
4. **Water in soils:** Infiltration and redistribution, vadose zone and soil moisture, ground water in hydrologic cycle.
5. **Rainfall and runoff:** Relations between rainfall and runoff, source of stream flow, excess rainfall and direct runoff. Abstraction using infiltration equation, SCS method for abstraction,  $\Phi$  index method, travel time, stream flow.
6. **Hydrograph:** Definition of hydrograph, unit hydrograph, synthesized unit hydrograph, applications of hydrographs.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. P. Jaya Rami Reddy, A text book of Hydrology.
2. V. Subramaniya, Engineering Hydrology.
3. Rafael L. Bras, Hydrology.
4. H. M. Raghunath, Hydrology.
5. V. P. Sing, Elementary Hydrology.

## AMTH 405: Differential Geometry and Tensor Analysis

4 credits

### Introduction and Specific Objectives

This course is a bridge between vector calculus and differential geometry, the intrinsic mathematics of curved spaces. The course initiates to the study of extrinsic geometry (curves and surfaces in  $n$ -space) to the intrinsic geometry of manifolds. Theory of tensors is a classical subject in multi-linear algebra, differential geometry and general relativity.

### Learning Outcomes

At the end of the course, the student will be able to apply mathematical concepts in the general theory of Relativity, Riemannian geometry and Engineering. Tensors are multi-arrays with at least 3 indices and it appears frequently in data analysis, communication and video surveillance. The recent interest in tensor in computer science is in quantum computing and quantum information theory.

## Course Contents

### Part A: Differential Geometry

1. **Curves in Space:** Vector functions of one variable and two variables, space curves, arc length, Tangent, Osculation plane, Normal, Principal normal, Binormal and fundamental planes. Curvature and torsion, Serret Frenet formula, Helics and their properties, Involute and Evolute.
2. **Surface:** Parametric curves, Tangent plane, normal and envelope, two and three parameter family of surfaces, First and second fundamental forms, Direction coefficients, orthogonal trajectories, Double family of curves. Curvature and directions, Rodrigue's formula, Euler's theorem.
3. **Geodesics:** Definitions, Differential equation of geodesics, geodesics on plane, sphere, right circular cone, cylinder, geodesic on a surface of revolutions.

### Part B: Tensor Analysis

4. **Vectors, Tensors and Co-ordinate transformations:** Kronecker delta, Covariant and contravariant vectors, Mixed and invariant tensors, addition, subtraction and multiplication of tensors, contraction, symmetric and skew-symmetric tensors, Quotient Law.
5. **Riemannian Metric and Metric Tensors:** Conjugate and associated tensors. Christoffel's symbols and their transformation laws.
6. **Covariant Differentiations of Tensors:** Covariant derivative of a tensor and higher rank tensors, intrinsic derivative, Tensor forms of gradient, divergence, curl and Laplacian, Riemann Christoffel tensor, Curvature tensor, Ricci tensor, Bianchi identity, Flat space and Einstein space and Applications.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 4 hours) 70 Marks. **Eight** questions of equal value will be set, **four** from each part, of which **five** are to be answered, taking at least **two** questions from each **Part**.

### References

1. T.J. Willmore, An Introduction to Differential Geometry.
2. S. Stamike, Differential Geometry.
3. M.M. Lischutz, Theory and Problems of Differential Geometry.
4. B. Spain, Tensor Calculus.
5. M. R. Spiegel, Vector and Tensor Analysis.

### AMTH 406: Asymptotic Analysis and Perturbation Methods

**3 credits**

#### Introduction and Specific Objectives

The development of the theory of asymptotic expansions serves as a foundation for perturbation methods and it is regarded as one of the most important achievements in applied mathematics in the twentieth century. Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. This course will introduce a range of modern asymptotic techniques and illustrate their use in model problems involving ordinary and partial differential equations.

#### Learning Outcomes

At the end of the course the students will be able to understand fundamental ideas used in the theory of asymptotic expansions. They will be able to develop appropriate practical skill in applying asymptotic methods for analyzing mathematical and physical problems with small or large parameters. The students will be introduced to the basic idea of singular perturbation theory.

#### Course Contents

1. **Asymptotic equivalence:** Asymptotic expansions. Taylor expansion as a conventional converging power series and as an example of an asymptotic expansion. Asymptotic expansions for definite integrals with the upper or lower limits of integration depending on small or large parameters. Functions defined by real integrals. Laplace's method for definite integrals, Watson's Lemma. Generalisation for functions defined by contour integrals. Steepest descent.
2. **Approximate solution of linear differential equations:** Asymptotic solutions of second-order linear equations (expansions near an irregular singularity, expansion for large arguments, equations containing a large parameter, equations involving a small parameter).
3. **Singular perturbations:** Method of strained coordinates, Inner and outer solutions. Overlap region. Matching of the asymptotic expansions. Ordinary differential equations with singular perturbations.
4. **Method of multiple scales:** Quasi-periodic solutions of second order ordinary differential equations developing non-uniformity at large time. Uniformly valid solutions. Amplitude equations. WKB Method.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. C. M. Bender and S.A. Orzag, Advanced Mathematical Methods for Scientists and Engineers, Springer.
2. J.D. Murray, Asymptotic Analysis. Springer.
3. E. J. Hinch, Perturbation Methods. Cambridge University Press.
4. F. W. J. Olver, Asymptotics and Special Functions.
5. Ali Hassan Nayfeh, Introduction to Perturbation Techniques.

## AMTH 407: Stochastic Calculus

**3 credits**

### Introduction and Specific Objectives

Stochastic calculus is used in financial engineering. We will cover the minimum of required math: sigma- algebras, conditional expectations, martingales, Wiener process, stochastic integration. The goal of this course is the Black and Scholes model and option pricing using martingale approach. This course is aimed at students with no measure theory background. We will formulate all the required theorems mostly without proofs. The only prerequisite for this course is probability theory: students should know how to calculate expectations, probabilities and conditional probabilities in discrete and continuous cases.

The course unit aims to provide the basic knowledge necessary to pursue further studies/applications where stochastic calculus plays a fundamental role (e.g. Financial Mathematics). The stochastic integral (Ito's integral) with respect to a continuous semimartingale is introduced and its properties are studied. The fundamental theorem of stochastic calculus (Ito's formula) is proved and its utility is demonstrated by various examples. Stochastic differential equations driven by a Wiener process are studied.

### Learning Outcomes

On successful completion of this course unit students will be able to

- understand the following mathematical concepts with their properties:
  - sigma-algebra
  - expectation w.r.to sigma algebra
  - martingale
  - Wiener process
  - the stochastic integral (Ito's integral)
- apply Ito's formula to smooth functions of continuous semimartingales;
- know basic facts and theorems of stochastic calculus;
- understand the concept of the stochastic differential equation driven by a Wiener process.

### Course Contents

1. **The Wiener process (standard Brownian motion):** Review of various constructions. Basic properties and theorems. Brownian paths are of unbounded variation.
2. **The Ito's integral with respect to a Wiener process:** Definition and basic properties. Continuous local martingales. The quadratic variation process. The Kunita-Watanabe inequality. Continuous semi-martingales. The Ito's integral with respect to a continuous semi-martingale: Definition and basic properties. Stochastic dominated convergence theorem.
3. **The Ito's formula:** Statement and proof. Integration by parts formula. The Levy characterization theorem. The Cameron-Martin-Girsanov theorem (change of measure). The Dambis-Dubins-Schwarz theorem (change of time).
4. **The Ito-Clark theorem:** The martingale representation theorem. Optimal prediction of the maximum process.

5. **Stochastic differential equations:** Examples: Brownian motion with drift, geometric Brownian motion, Bessel process, squared Bessel process, the Ornstein-Uhlenbeck process, branching diffusion, Brownian bridge. The existence and uniqueness of solutions in the case of Lipschitz coefficients.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Rogers, L. C. G. and Williams, D., Diffusions, Markov Processes and Martingales.
2. Revuz, D. and Yor, M., Continuous Martingales and Brownian Motion.
3. Karatzas, I. and Shreve, S. E., Brownian Motion and Stochastic Calculus.
4. Durrett, R., Stochastic Calculus.

## AMTH 408: Econometrics

**3 credits**

### Introduction and Specific Objectives

Econometrics deals with the measurement of economic relationships. It is an integration of economics, mathematical economics and statistics with an objective to provide numerical values to the parameters of economic relationships. The relationships of economic theories are usually expressed in mathematical forms and combined with empirical economics. The econometrics methods are used to obtain the values of parameters which are essentially the coefficients of mathematical form of the economic relationships. The statistical methods which help in explaining the economic phenomenon are adapted as econometric methods. The econometric relationships depict the random behaviour of economic relationships which are generally not considered in economics and mathematical formulations.

### Learning Outcomes

To analyze critically the basic elements of Econometrics in order to understand the logic of econometric modelling and be able to specify causal relationships among economic variables. To identify the relevant statistical sources in order to search for, organize and systematically arrange available economic data. To use with confidence appropriate statistical methods and available computing tools in order to correctly estimate and validate econometric models. To handle econometric prediction tools in order to estimate unknown or future values of an economic variable. To interpret adequately the results obtained in order to be able to write meaningful reports about the behaviour of economic data.

### Course Contents

1. **The Nature of Econometrics and Economic Data:** Econometrics, Steps in Empirical Economic Analysis, The Structure of Economic Data, Causality and the Notion of Ceteris Paribus in Econometric Analysis.
2. **The Simple Regression Model:** Definition of the Simple Regression Model, Deriving the Ordinary Least Squares Estimates, Properties of OLS on Any Sample of Data, Units of Measurement and Functional Form, Expected Values and Variances of the OLS Estimators, Regression through the Origin.
3. **Multiple Regression Analysis:** Estimation: Motivation for Multiple Regression, The Model with Two Independent Variables, The Model with  $k$  Independent Variables, Mechanics and Interpretation of Ordinary Least Squares, The Expected Value of the OLS Estimators, The Variance of the OLS Estimators, Efficiency of OLS: The Gauss-Markov Theorem.
4. **Multiple Regression Analysis:** Inference: Sampling Distributions of the OLS Estimators, Testing Hypotheses about a Single Population Parameter: The  $t$  Test, Confidence Intervals, Testing Hypotheses about a Single Linear Combination of the Parameters, Testing Multiple Linear Restrictions (The  $F$  Test), Reporting Regression Results.

5. **Multiple Regression Analysis:** OLS Asymptotics: Consistency, Deriving the Inconsistency in OLS , Asymptotic Normality and Large Sample Inference, Asymptotic Efficiency of OLS, Effects of Data Scaling on OLS Statistics(Beta Coefficients ), More on Functional Form( Logarithmic Functional Forms, Models with Quadratics, Models with Interaction Terms )
6. **Basic Regression Analysis with Time: Series Data:** The Nature of Time Series Data, Some of Time Series Regression Models: Static Models , Finite Distributed Lag Models, A Convention about the Time Index, Finite Sample Properties of OLS under Classical Assumptions, Functional Form, Dummy Variables, and Index Numbers, Trends and Seasonality.
7. **Further Issues in Using OLS with Time Series Data:** Stationary and Weakly Dependent Time Series, Stationary and Nonstationary Time Series, Weakly Dependent Time Series , Asymptotic Properties of OLS, Using Highly Persistent Time Series in Regression Analysis, Highly Persistent Time Series, Transformations on Highly Persistent Time Series, Deciding Whether a Time Series Is  $I(1)$  , Dynamically Complete Models and the Absence of Serial Correlation.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. Jeffrey M. Wooldridge, Introductory Econometrics: A Modern Approach.
2. Joshua D. Angrist & Jörn-Steffen Pischke, Mostly Harmless Econometrics: An Empiricist's Companion.
3. Johnston J. and DiNardo, J., Econometric Methods.

## AMTH 409: Actuarial Mathematics

**3 credits**

### Introduction and Specific Objectives

Actuarial Mathematics course is designed to equip the students with training in theoretical and practical aspects of Actuarial Science in order for them to work in life and non-life insurance companies (designing insurance products and valuing financial contracts and investing funds); consultancy (offering advice to occupational pension funds and employee benefit plans); government service (supervising insurance companies and advising on the national insurance); and also in the stock exchange, industry, commerce and academia. These problems involve analyzing future financial events, especially where future payments involved have certain or uncertain timing. The traditional areas in which actuaries operate are: life and general insurance, pensions, and investment. Actuaries are also increasing moving into other fields like health insurance, solvency measurements and asset liability management, financial risk management, mortality and morbidity investigation and so on, where their analytical skills can be employed. Currently, there is high demand for Actuarial Science expertise locally, regionally and internationally.

### Learning Outcomes

Students will

- be able to demonstrate a solid foundation in mathematics by their ability to solve a variety of basic and advanced mathematical problems.
- be able to correctly solve a wide variety of actuarial science problems using both basic and advanced mathematical techniques.
- learn to apply actuarial mathematics to problems in a variety of fields, including insurance, finance, investment, and other businesses.
- communicate effectively and clearly both in written and oral forms.
- apply technology to actuarial problem solving.
- gain practical experience in actuarial science through internships, independent projects, or research.



## Course Contents

1. **Survival models:** Survival models, Some actuarial concepts in survival analysis, Force of Mortality, Expectation of life, Curtate failure, Selected survival models, Common Analytical Survival Models, Mixture models.
2. **Life Tables:** Life tables, Actuarial Models, Deterministic survivorship group and random survivorship group, Continuous computations, Interpolating life tables, Select and Ultimate Tables.
3. **Life insurance:** Introduction to life insurance, Payments paid at the end of the year of death. Further properties of the APV for discrete insurance, Non-level payments paid at the end of the year, Payments at the end of the m-thly time interval, Level benefit insurance in the continuous case. Further properties of the APV for continuous insurance, Non-level payments paid at the end of the year, Computing APV's from a life table.
4. **Life annuities:** Whole life annuity, n-year deferred annuity, n-year temporary annuity, n-year certain annuity, Contingencies paid m times a year, Non-level payments annuities, Computing present values from a life table.
5. **Benefit premiums:** Funding a liability. Fully discrete benefit premiums. Benefits paid annually funded continuously. Benefit premiums for fully continuous insurance. Benefit premiums for semicontinuous insurance. Benefit premium for an n-year deferred annuity. Premiums paid m times a year. Non-level premiums and/or benefits. Computing benefit premiums from a life table, Premiums found including expenses.
6. **Benefit reserves:** Benefit reserves, Fully discrete insurance. Fully continuous insurance, Reserves for insurance paid immediately and funded discretely, Reserves for insurance paid discretely and funded continuously, Benefit reserves for general fully discrete insurance, Benefit reserves for general fully continuous insurance, Benefit reserves for m-thly paid premiums. Benefit reserves including expenses. Benefit reserves at fractional durations.
7. **Multiple life functions :** Multivariate random variables, Joint life status, Last survivor status, Joint survival functions, Common shock model, Insurance for multi-life models, Problems for recent actuarial exams,
8. **Markov chains:** Stochastic processes. Markov chains, Random walks, Hitting probabilities, Gambler's ruin problem, Some actuarial applications.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

## References

1. S. David Promislow – Fundamentals of Actuarial Mathematics.
2. Newton L. Bowers, Hans U. Gerber – Actuarial Mathematics, Society of Actuaries.
3. <http://www.math.binghamton.edu/arcones/450/syllabus.html>.

## AMTH 410: Heat Transfer

**3 credits**

### Introduction and Specific Objectives

This course is designed to introduce a basic study of the phenomena of heat transfer, to develop methodologies for solving a wide variety of practical application in industry, and to provide useful information concerning the performance and design of particular systems and processes. Basic physics and methodology to understand heat transfer are covered. This course covers the development of the fundamental equations of conduction, radiation and convection heat transfer mechanisms. Topics include Basics of Heat Transfer, Heat Conduction, Numerical Methods in Heat Conduction, Fundamentals of Thermal Radiation, Fundamentals of Convection, and Natural Convection. The objectives of this course are to give a general overview to students about applying of scientific and engineering principles to analyze and design aspects of engineering systems that relate to conduction, radiation and convection heat transfer, using appropriate analytical and computational tools to investigate conduction, radiation and convection heat transfer.

### Learning Outcomes

After successful completion of this course, the students will be able to understand the basic laws of heat transfer, the basic mechanisms of heat transfer, the general heat diffusion equation. The learners will be able to model and solve different types of heat conduction problems. The students will be able to understand the fundamentals of thermal and solar radiation. Besides, they will understand the physical phenomena associated with convection, Newton's law of cooling, and the significance of nondimensional parameters in convection heat transfer. Finally, the students will learn the governing equations, physical mechanism associated with natural convection and its applications over surfaces and inside enclosures.

### Course Contents

1. **Basics of Heat Transfer:** Thermodynamics and Heat Transfer; Heat and Other Forms of Energy; The First Law of Thermodynamics; Heat Transfer Mechanisms: Conduction; Convection and Radiation; Simultaneous Heat Transfer Mechanisms. Thermal Insulation.
2. **Heat Conduction:** Introduction; The conduction equation; Steady state conduction in Simple Geometries. Extended surfaces; Multidimensional Steady Conduction; Transient Heat Conduction.
3. **Numerical Methods in Heat Conduction:** Why Numerical Methods? Numerical solution of One-Dimensional Steady Heat Conduction; Two-Dimensional Steady Heat Conduction and Transient Heat Conduction Problems.
4. **Fundamentals of Thermal Radiation:** Introduction; Thermal Radiation; Blackbody Radiation; Radiative properties; The Green House Effect; Atmospheric and Solar Radiation.
5. **Fundamentals of Convection:** Physical mechanism of Convection; Classification of Fluid Flows; Velocity Boundary Layer; Thermal Boundary Layer; Laminar and Turbulent Flows; Derivation of Differential Convection Equations; Nondimensionalized Convection Equations and Similarity. Forced and Natural Convections.
6. **Natural Convection:** Introduction; Boussinesq Approximation; Physical mechanism of Natural Convection; Equation of motion and the Grashof Number. Natural Convection over surfaces. Natural Convection inside Enclosures.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. J. P. Holman, Heat Transfer.
2. Frank Kreith, Raj M. Manglik, Mark S. Bohn, Principles of Heat Transfer.
3. Yunus A. Cengel, Heat Transfer, A Practical Approach.

## AMTH 411: Modern Astronomy

**3 credits**

### Introduction and Specific Objectives

This course introduces the science of modern astronomy with a concentration on the solar system. Emphasis is placed on the history and physics of astronomy and an introduction to the solar system, including the planets, comets, and meteors.

### Learning Outcomes

Upon completion, students should be able to demonstrate a general understanding of the solar system. Demonstrate conceptual understanding of fundamental physical principles and measurement techniques used in modern astronomy. Demonstrate conceptual understanding of astronomical objects and phenomena. Understand various objects within the Solar System. Understand the size, scale, and structure of the Solar System. Be able to observe the sky and gain an understanding of the objects and motions visible.

### Course Contents

1. **Celestial Sphere:** Sphere and spherical triangles, the celestial sphere, problems connected with diurnal motion.
2. **Astronomical Co-ordinate:** The first system of coordinates, The Second system of coordinates, The Third system of coordinates, Transformation Co-ordinates, Astronomical Refraction, The ecliptic and the first point of Aries.
3. **Kepler's laws:** Equations of time, unit of time.
4. **Geocentric parallax:** The moon, Local line, Eclipses.
5. **The Solar System:** Planets, Bode's Law, Sidereal Period and synodic period of a Planet, General Description of Solar System.
6. **The Moon:** Moon's Librations, Relation between Sidereal months and synodic months, Phases of Moon, Moon's Nodes and Nodal period, Daily retardation of moon-rise.
7. **Precession and nutation:** Annual parallax, Aberration of light.
8. **The stellar universe:** Modern finding of astronomical objects, Working process of the Hubble telescope and its finding.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. K.R. Khan & A.Z. Sikder, Astronomy.
2. W.R. Smart, Spherical Astronomy.
3. G.V. Ramchandran, A Text Book of Astronomy.

## AMTH 412: Quantum Theory and Special Relativity

**3 credits**

### Introduction and Specific Objectives

This course aims to introduce students to field quantisation, scalar field theory and quantum electrodynamics and to develop calculational techniques in perturbation theory. Special Relativity is probably the most misunderstood theory of physics. Fewer people know anything about General Relativity or Quantum Mechanics, but for no other theory in physics have so many people been told so many things about the theory that are philosophically important and wrong. It is widely agreed that Einstein taught us that "all things are relative" and "nothing is absolute." Neither of these are any part of Special Relativity, which simply gives transformation rules that allow one person to translate his measurements so that they will *agree* with the measurements of other, moving, people. We need some of these transformation rules for particle physics, especially the rules about energies, and we might as well learn a little about the conceptual aspects of special relativity while we are at it.

### Learning Outcomes

On completion of the course, students should understand field quantisation and the expansion of the scattering matrix and to be able to carry out practical calculations based on Feynman diagrams. Understand how to construct and use space-time tensor equations to solve practical problems in applied physics, determine under what conditions relativistic effects such as time dilation and Einstein velocity addition are important and calculate the pseudo acceleration caused by coordinate transformations.

### Course Contents

1. **Wave-particle duality:** Schrödinger's equation; stationary states; quantum states of a particle in a box, infinite square well potential, finite square wells, boundary conditions at a potential step, bound states in a finite well, reflection and transmission by a finite step, and by a barrier, tunnelling.
2. **The one-dimensional harmonic oscillator:** higher-dimensional oscillators and normal modes; degeneracy.

3. **The basic postulates of quantum mechanics:** Commutation relations and compatibility of different observables; Heisenberg's uncertainty principle.
4. **Angular momentum in quantum mechanics:** angular momentum operators; Orbital angular momentum, particle in two dimensions (eigenfunctions and eigenvalues of  $L_z$ ), particle in three dimensions (eigenfunctions and eigenvalues of  $L^2$  and  $L_z$ ), rotational states of a diatomic molecule; Spherical harmonics.
5. **Hydrogen Atom:** Central potential, Energy levels, size and shape of energy eigenfunctions, effect of finite mass of nucleus, EM spectrum, hydrogen-like systems.
6. **Electron spin:** Stern-Gerlach experiment, quantum states of two identical particles, spin and space wave functions and origin of the Pauli Exclusion Principle. Energy states of the He atom.
7. **Constancy of the speed of light:** Galilean relativity, Maxwell's equations, wave equation in electromagnetism, Principles of Einstein's special relativity, Lorentz transformations, time dilation, length contraction, simultaneity, space-time separation, the Twin paradox, causality.
8. **Tensor equations:** Index notation, four-vectors, four-velocity and four-momentum; equivalence of mass and energy:  $E = mc^2$ ; particle collisions and four-momentum conservation; Photons and Compton scattering, mass transport by photons, particle production and decay, four-acceleration and four-force, Lorentz force, the example of the constant-acceleration world-line, the relativistic Doppler effect.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

### References

1. B.H. Bransden and C.J Joachain Quantum Mechanics.
2. P.C.W. Davies and D.S. Betts, Quantum Mechanics.
3. R.P Feynman, R.B Leighton, M. Sands The Feynman Lectures on Physics.
4. A.I.M. Rae, Quantum Mechanics.
5. N. M. J. Woodhouse, Special Relativity.
6. W Rindler, Introduction to Special Relativity.

## AMTH 413: Mathematical Modeling in Biology and Physiology

**3 credits**

### Introduction and Specific Objectives

The life sciences are arguably the greatest scientific adventure of the age. Over the last few decades a series of revolutions in experimental technique have made it possible to ask very detailed questions about how life works, ranging from the smallest, sub-cellular scales up through the organization of tissues and the functioning of the brain and, on the very largest scales, the evolution of species and ecosystems. Mathematics has so far played a small, but honorable part in this development, especially by providing simple models designed to illuminate principles and test broad hypotheses.

The objective of the course is an introduction to the mathematical modeling of biological processes, with emphasis on population biology including ecology, biochemistry and physiology with technique include difference equations, ordinary differential equations, partial differential equations, stability analysis, phase plane analysis.

**Learning Outcomes:** The students will be able to

- use simple ODE models to discuss questions in population dynamics.
- read, understand and analyze dynamical systems that describe networks of biochemical reactions.
- gain the ability of understanding the Hodgkin–Huxley Model and the FitzHugh–Nagumo model

### Course Contents

1. **Introduction to modeling in Biology:** Mathematical modeling and Mathematical modeling in Biology. Basic idea to create a model in Biology.

2. **Analysis of Dynamic Mathematical Models (Discrete and Continuous):** Graphical Analysis, Linearization and Bifurcation for First Order Differential Equations and Phase Plane Analysis for Second Order Differential Equations.
3. **Population Models for single species:** Discrete Population Models: Introduction: Simple Models, Logistic-Type Model, Fishery Management Model and Delay Models. Continuous population model: Simple Model, Logistic Model, Insect Outbreak Model, Harvesting a Single Natural Population and Delay Models.
4. **Population Dynamics of Interacting Species:** Host-parasitoid Interactions, Predator–Prey Models: Lotka–Volterra Systems, Competition, Mutualism or Symbiosis.
5. **Infectious Diseases:** The Simple Epidemic and SIS Diseases, SIR Epidemics, SIR Endemics- No Disease-related Death and Including Disease-related Death.
6. **Modelling on Chemical Reaction Network:** Closed and Open networks, Dynamic behavior of reaction networks, and Numerical simulation of differential equations.
7. **Excitability:** The Hodgkin–Huxley Model- History of the Hodgkin–Huxley Equations, Voltage and Time Dependence of Conductance's, Qualitative Analysis, The FitzHugh–Nagumo Equations- The Generalized FitzHugh–Nagumo Equations, Phase-Plane Behavior.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

#### References

1. J.D. Murray, Mathematical Biology: I. An Introduction
2. Nicholas F. Britton, Essential Mathematical Biology
3. James Keener and James Sneyd, Mathematical Physiology I.
4. S. J. Chapman, A. C. Fowler & R. Hinch, An Introduction to Mathematical Physiology

#### References

1. G. Bard Ermentrout, David H. Terman, Mathematical Foundations of Neuroscience.
2. Alla Borisjuk, Avner Friedman, Bard Ermentrout, David Terman - Tutorials in Mathematical Biosciences I.
3. Peter Dayan, L. F. Abbott -Theoretical Neuroscience Computational and Mathematical Modeling of Neural Systems-The MIT Press (2005).
4. H. Wilson - Spikes, Decisions and Actions, Visual Sciences Center, University of Chicago.

#### AMTH 414: Biomechanics

**3 credits**

#### Objective:

Biomechanics is the study of how physical forces interact with living system. It provides a broad overview of the rapidly growing field of bioengineering, ranging from the mechanics of single cells to the dynamics of human movement. This course provides students with a deeper understanding of both the mechanics and the biology .

Outcomes:

- 1] Understand the physiology and anatomy of the studied systems,
- 2] Analyze mathematical and mechanical models currently used for clinical research problems,
- Integrate engineering concepts to examine and to model the biological flow in human body,
- Develop a critical thinking regarding the current research challenges in human movement.

#### **Contents:**

1. Introduction:  
A brief history of biomechanics
2. Cellular biomechanics
- 2.1 Cell–matrix interactions

- 2.2 Methods to measure the mechanical properties of cells and biomolecules
- 2.3 Models of cellular biomechanical behavior
- 2.4 Mechanotransduction: how do cells sense and respond to mechanical events?
- 2.5 Techniques for mechanical stimulation of cells
- 2.6 Problems

### 3 Hemodynamics

- 3.1 Blood rheology
- 3.2 Large artery hemodynamics
- 3.3 Blood flow in small vessels
- 3.4 Problems

### 4 The circulatory system

- 4.1 Anatomy of the vasculature
- 4.2 The heart
- 4.3 Arterial pulse propagation
- 4.4 The capillaries
- 4.5 The veins
- 4.6 Scaling of hemodynamic variables
- 4.7 Problems

### 5 The interstitium

- 5.1 Interstitial fluid flow
  - 5.1.1 Darcy's law
  - 5.1.2 Clearance of edema
- 5.2 Problems

### 6 Ocular biomechanics

- 6.1 Ocular anatomy
- 6.2 Biomechanics of glaucoma
- 6.3 Ocular blood flow
- 6.4 Problems

### 7 The respiratory system

- 7.1 Biomechanics of breathing
- 7.2 Lung elasticity and surface tension effects
- 7.3 Mass transfer
- 7.4 Particle transport in the lung
- 7.5 Problems

#### **Suggested Reading:**

- 1] Introductory Biomechanics From Cells to Organisms, C. Ross Ethier and Craig A. Simmons, Cambridge University Press, 2007.
- 2] Biomechanics: Circulation (Fung, Y. C., Springer Verlag, 1996
- 3] Biofluid Mechanics (Mazumdar, J. N., World Scientific, Inc., 1992

### **AMTH 415: Industrial Mathematics**

### **3-Credits**

#### **Introduction and Specific Objectives**

Mathematics is unreasonably effective in resolving seemingly intractable problems. The process proceeds in : model the external world problem as a mathematical problem, solve the mathematical problem, then interpret the results. Students in government or industry will be involved at all the three steps through this course.

**Learning Outcomes:** Students in industry must be able to see their work from an economic viewpoint.

#### **Course Contents**

- 1. Statistical reasoning:** Random variables, Uniform distributions, Gaussian distributions, The binomial distribution, The Poisson distribution, Taguchi quality control.
- 2. Data acquisition and manipulation:** The z-transform, Linear recursions, Filters, Stability, Polar and Bode plots, Aliasing, Closing the loop, Why decibels?
- 3. Cost benefit analysis:** Present value, Life cycle costing.
- 4. Microeconomics:** Supply and demand, Revenue, cost, and profit; Elasticity of demand, Duopolistic competition, Theory of production, Leontiev input/output.
- 5. Frequency domain methods:** The frequency domain, Generalized signals, Plants in cascade, Surge impedance, Stability, Filters, Feedback and root-locus, Nyquist analysis, Control.
- 6. Divided differences:** Euler's method, Systems, PDEs, Runge-Kutta
- 7. Splines:** Why cubics? m-Splines, Cubic splines

**Evaluation:** In course Assessment 30 marks, Final examination (Theory, 3 hours) 70 Marks.

Eight questions of equal value will be set, of which any five are to be answered.

#### **Reference**

1. C. R. MacCluer , Industrial Mathematics: modeling in industry, science, and government, Prentice Hall, 2000

### **AMTH 416: Computational Science and Engineering**

**3-Credits**

#### **Introduction and Specific Objectives**

This course serves all mathematicians and engineers and scientists to explain the core ideas of applied mathematics and computing.

#### **Learning Outcomes**

To bring ideas and algorithms together into a more useful course in a natural way to become computational science and engineering.

#### **Course Contents**

- 1. Applied Linear Algebra:** Four Special Matrices, Differences, Derivatives, and Boundary Conditions, Elimination Leads to  $K = LDL^T$ , Inverses and Delta Functions, Positive Definite Matrices, Numerical Linear Algebra: LU, QR, SVD.
- 2. A Framework for Applied Mathematics:** Equilibrium and the Stiffness Matrix, Oscillation by Newton's Law , Least Squares for Rectangular Matrices , Graph Models and Kirchhoff's Laws , Networks and Transfer Functions , Nonlinear Problems, Structures in Equilibrium, Covariances and Recursive Least Squares, Graph Cuts and Gene Clustering.
- 3. Boundary Value Problems:** Differential Equations of Equilibrium , Cubic Splines and Fourth Order Equations, Finite Differences and Fast Poisson Solvers, Elasticity and Solid Mechanics .
- 4. Fourier Series and Integrals:** Convolution and Signal Processing, Deconvolution and Integral Equations , Wavelets and Signal Processing .
- 5. Analytic Functions:** Famous Functions and Great Theorems, The Laplace Transform and z-Transform, Spectral Methods of Exponential Accuracy.
- 6. Initial Value Problems:** Introduction, Finite Difference Methods for ODE's , Accuracy and Stability for  $u_t = c u_x$ , The Wave Equation and Staggered Leapfrog, Diffusion, Convection, and Finance, Nonlinear Flow and Conservation Laws, Fluid Mechanics and Navier-Stokes , Level Sets and Fast Marching .
- 7. Solving Large Systems:** Elimination with Reordering, Iterative Methods, Multigrid Methods, Conjugate Gradients and Krylov Subspaces.

**Evaluation:** In course Assessment 30 marks, Final examination (Theory, 3 hours) 70 Marks.

Eight questions of equal value will be set, of which any five are to be answered.

#### **References:**

1. Gilbert Strang, Computational Science and Engineering, 2006.
2. Gilbert Strang, Introduction to Applied Mathematics, 2009.

**AMTH 430: Special Topics****3 credits**

Any mathematical topic not covered in other courses may be offered under this title. The course-teacher will prepare an outline of the course and obtain the approval of the departmental academic committee.

**Evaluation:** Incourse Assessment and Attendance 30 Marks, Final examination (Theory: 3 hours) 70 Marks. Eight questions of equal value will be set, of which any five are to be answered.

**AMTH 450: Math Lab IV (Application software)****3 credits**

Problem solving in concurrent courses on First year to Fourth year using different application software imposed by the lab teachers.

**Lab Assignments:** There are at least 07 assignments.

**Evaluation:** Internal assessment (Laboratory works) 40 Marks. Final examination (Lab, 3 hours) 60 Marks.

**AMTH 460: Project and Presentation****3 credits****Introduction and Specific Objectives**

Each student is required to work on a project and present a project report for evaluation. Such projects should be extensions or applications of materials included in different honours courses and may involve field work and use of technology. There may be group projects as well as individual projects. The Academic Committee shall form a Project Coordination and Evaluation Committee (PCEC) at the beginning of the session. The PCEC shall consist of a project Coordinator (PC) and members as the Academic Committee considers appropriate. The PC shall invite projects from the teachers before the class start. Each teacher should submit three project proposal should include a short description of the project. Such projects should be extension or applications of materials included in different courses, and may involve fieldwork and use of technology.

The PCEC shall assign each group a project. The members of each group shall work independently on the assigned project under the supervision of the concerned teacher. The PCEC and the supervisors will monitor the progress of different projects.

**Completion of project**

The project must be completed before the termination of the classes. Each student is required to prepare a separate report on the project. Each report should be of around 40 pages typed on one side of A4 size white paper preferably using word processors. Graphs and figures should be clearly drawn preferably using computers. Reports of different students working on the same group project should differ in some details and illustrations.

The Academic Committee will select a date for the submission of the project reports to the PCEC. Each student must submit three printed copies of her/his project report to the PCEC on or before the date announced for such submission.

The PCEC, on receiving the reports will arrange the presentation of reports by individual students before the PCEC. This presentation should take place soon after the completion of the written examination.

Any student who fails to submit the report on the due date or to present the thesis on the fixed date will not get any credit for this course.

**Evaluation**

The distribution of marks for each project shall be as follows:

Project Report                      60 marks.

Project Presentation              40 marks.

Each project report shall be examined by two examiners, one of whom shall be the project supervisor and the other appointed from amongst the teachers of the department on the recommendation of the PCEC. In case the marks of the two examiners of a project report differ by more than 20% a third



examiner for that report shall be appointed from amongst the teachers of the department on the recommendation of the PCEC. In such cases the final marks shall be determined according to the usual procedure. Each student is required to present her/his work on the project before the PCEC who will evaluate the presentation. All marks on the projects shall be submitted to the Examination Committee for tabulation with copies to the Controller of Examinations. The project reports shall be returned to the PCEC for preservation.

**AMTH 499: Viva Voce**

**2 credits**

Viva voce on courses taught in Fourth Year.