Curriculum for One-Year M. S. Program
Department of Mathematics
University of Dhaka
(Effective for 2019-2020, 2020-2021)

The Department offers M. S. (Masters of Science) program in Mathematics. The M. S. program has duration of one academic year. In the program there is provision for taking up thesis work, subject to the approval of the academic committee of the department.

Each student in **Group A** (**Non-Thesis Group**) of the program has to take **seven** courses (each of 4 credits), while each student in **Group B** (**Thesis Group**) has to take **six** courses, out of the courses enumerated below, subject to conditions laid down by the academic committee.

Group A (Non-Thesis Group) **32 Credits**
Credit Requirement

7 Courses, 28 Credits
Viva Voce 4 Credits

Group B (Thesis Group) **36 Credits**
Credit Requirement

6 Courses, 24 Credits
Thesis 8 (6+2) Credits
Viva Voce 4 Credits

List of Courses for M. S. Program

MTMS 501	THEORY OF GROUPS	4 Credits
MTMS 502	THEORY OF RINGS AND MODULES	4 Credits
MTMS 503	ADVANCED NUMBER THEORY	4 Credits
MTMS 504	REAL FUNCTION THEORY	4 Credits
MTMS 505	COMPLEX FUNCTION THEORY	4 Credits
MTMS 506	GENERAL TOPOLOGY	4 Credits
MTMS 507	FUNCTIONAL ANALYSIS	4 Credits
MTMS 508	LIE GROUPS AND LIE ALGEBRAS	4 Credits
MTMS 509	FUZZY MATHEMATICAL STRUCTURES	4 Credits
MTMS 510	SPECIAL TOPICS	4 Credits
MTMS 511	DIFFERENTIAL AND INTEGRAL EQUATIONS	4 Credits
MTMS 512	OPERATIONS RESEARCH	4 Credits
MTMS 513	NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS	4 Credits
MTMS 514	GEOMETRY OF DIFFERENTIAL MANIFOLDS	4 Credits
MTMS 515	DYNAMICAL SYSTEMS	4 Credits
MTMS 516	MATHEMATICAL BIOLOGY	4 Credits
MTMS 517	OPERATIONS MANAGEMENT	4 Credits
MTMS 518	ASYMPTOTIC AND PERTURBATION THEORY	4 Credits
MTMS 519	QUANTITATIVE FINANCIAL RISK MANAGEMENT	4 Credits
MTMS 520	SPECIAL TOPICS	4 Credits
MTMS 590	MS THESIS	8 Credits
MTMS 599	VIVA VOCE	4 Credits

MTMS 501: THEORY OF GROUPS **Credits: 4**

Rationale:

The main emphasis of this course is on Finite Groups and classification of Groups of small order. However, results will be stated for infinite groups too, wherever possible. The goal of this course is to teach Conjugacy Classes, Permutation Groups and Symmetry Groups and their applications in real life, Sylow Theorems and techniques of their proofs, with application in Finite Groups; Direct Products, Extension of Groups; Factor Groups, Nilpotent, Soluble and Supersoluble Groups; Group Representations and Characters of Groups.

***Course Objectives*:**

By the end of the module, students should be familiar with the topics listed in the Course Contents. In particular, students will be able to prove the Class Equation for finite Groups, learn the techniques to prove Sylow Theorems and their applications for analyzing the structures of Finite Groups of given orders. They should be able to find Extensions and Split Extensions of groups; find Representation using Matrix; prove Schur's Lemma, Maschke's Theorems; find Group Characters.

Course Contents:

1. Centralizer, Normalizer in a Group; Characteristic subgroup; classification of Groups of small orders, Lagrange's Theorem and its falsity
2. Symmetry Groups, Alternating Groups, Dihedral Groups
3. Permutation Groups, cycles, orbits, transitivity, representation of Group by Permutations
4. Sylow Theorems; Finite p-groups, classification of Groups of order p, p^2, pq, p^3
5. Automorphisms, inner Aotomorphisms
6. Group Extensions, Direct Products, Cyclic Extentions, Split Extentions, Semi-direct Product, Wreath Products
7. Soluble, Supersoluble, Nilpotent Groups and their subgroups; Commutator Group, Composition series, Normal series, Factor Groups; Upper and Lower Central Series
8. Permutational and Matrix Representation of Groups, Reducibility, Schur's Lemma, Maschk's Theorem
9. Group Characters, Reducible, Irreducible, Faithful Characters; Orthogonality of First and Second Kind.

***Learning Outcomes*:**

The main Learning Outcomes, after completing this module, will be

1. To understand basic ideas and applications of Groups
2. To get introduced to different terminologies and properties of Finite Groups
3. To get familiar with different classes of Groups, such as Symmetry Groups, Permutation Groups, Dihedral Groups, Klein4 Groups
4. To find and prove the Class Equation for Finite Groups
5. To learn the techniques of proofs of Sylow Theorems in the module

6. To learn to apply Representation Theory of Groups and decomposition into irreducible representations to find Group Characters of Finite Groups.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. Marshall Hall, Jr, The Theory of Groups, Literary Licensing
2. W. Ledermannn, Introduction to Group Theory, Longman
3. Martin Burrow, Representation Theory of Finite Groups, Dover Publications
4. W. Ledermann, Introduction to Group Characters, Cambridge University Press
5. B. Baumslag and B. Chandler, Schaum's Outline of Group Theory, McGraw-Hill
6. I. D. Macdonald, The Theory of Groups, Oxford University Press
7. Thomas W. Judson, Abstract Algebra: Theory and Application.

MTMS 502: THEORY OF RINGS AND MODULES **Credit: 4**

Rationale:

A ring is an important fundamental concept in algebra and includes integers, polynomials and matrices as some of the basic examples. Ring theory has applications in number theory and geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring ***R*** provides us with an insight into the structure of ***R***. In this course we shall develop ring and module theory leading to the fundamental theorems of Wedderburn and some of its applications.

Course Objectives:

By the end of the course the student should understand

1. The importance of rings and modules as central objects in algebra and some of its applications.
2. The basic structure and theory of rings and modules.
3. How to develop this theory to investigate important classes of integral domains.
4. The concept of a module as a generalization of a vector space and an Abelian group.
5. The classification of any finitely generated module as a homomorphic image of a free module.
6. Simple modules, Schur's lemma. Radical, simple and semi simple artinian rings. Examples.
7. Semi-simple modules, artinian modules, their endomorphism. Examples.
8. The Wedderburn-Artin theorem.

Course Contents:

1. Rings of fractions and embedding theorems. Local rings and Noetherian rings.
2. Rings with Ore conditions and related theorems.
3. Field extensions and finite fields. Wedderburn’s theorem on finite dimension rings.
4. Jacobson’s work on commutativity of rings.
5. Modules, submodules and direct sums. R-homomorphisms and quotient modules.
6. Completely reducible and free modules.

7. Sequences and exact sequences of modules. Projective and injective modules.
8. Semisimple and simple rings.
9. Noetherian and Artinian modules. Wedderburn-Artin theorem.

Learning Outcomes:

Upon successful completion of this course students will be able to

1. Understand the central role of abstract algebra in modern mathematics.
2. See the relations between algebra and its applications in and outside mathematics.
3. Become familiar with rings and fields, and understand the structure theory of modules over a Euclidean domain along with its implications.
4. Write precise and accurate mathematical definitions of objects in ring theory.
5. Use mathematical definitions to identify and construct examples and to distinguish examples from non-examples.
6. Validate and critically assess a mathematical proof.
7. To understand how every finitely generated module is a homomorphic image of a free module.
8. Use a combination of theoretical knowledge and independent mathematical thinking to investigate questions in ring theory and to construct proofs.
9. Write about ring theory in a coherent, grammatically correct and technically accurate manner.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. Hiram Paley and Paul M. Weichsel, A First Course in Abstract Algebra, Holt, Rinehart and Winston
2. S. Lang, Algebra, Springer
3. Thomas W. Hungerford, Algebra, Springer
4. P. B. Bhattarcharya, S. K. Jain & S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press

MTMS 503: ADVANCED NUMBER THEORY Credits: 4

Course Content:

1. **Quadratic Residuacity:** Quadratic residues and nonresidues, Euler criterion, Legendre symbol, Gauss's lemma, law of quadratic reciprocity, Jacobi's symbol.
2. **Average Orders of Arithmetic Functions:** Lim sup, Lim inf, average orders of the arithmetical functions.
3. **Distribution of Prime Numbers:** Bertrand's postulate, Chebyshev's theorem, the function $\theta(x)$ and $\psi(x)$. The prime number theory; elementary proof via Selberg's lemma, complex analytical proof.
4. **Primes in Arithmetic Progressions:** Characters of an Abelian group, L-functions, Dirichlet's proof of infinitude of primes in arithmetic progressions.

5. **Algebraic Number Theory:** Noetherian rings and Dedekind domains, ideal classes and the unit theorem, units in real quadratic field.

***Rationale, Course Objectives and Learning Outcomes:** The course instructor will provide the details.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **Five** are to be answered.

References:

1. G. H. Hardy and E. M. Wright, An introduction to the theory of numbers, Oxford University Press
2. S. Rose, A course in Number Theory, Oxford University Press
3. P. Samuel, Algebraic Theory of Numbers, Dover Publications

MTMS 504: REAL FUNCTION THEORY **Credits: 4**

Rationale:

This course is an introduction to measure and integration theory, the theory of metric spaces, and their applications to the approximation of real valued functions. The course starts with notions of basic topological properties of Euclidean spaces, Algebra of sets, Cantor and Borel sets. Then we study convergence for sequences of continuous functions, the Ascoli-Arzela compactness theorem, and the Weierstrass approximation theorem.

***Course Objectives*:**

The main body of the course deals with the theory of measure and integration and limiting processes for the Lebesgue integral. The last part covers the topics of Differentiation, Functions of bounded variation and Fourier Series. The topics to be covered are

1. Review of continuous functions, Metric spaces, Sequences of functions, uniform convergence, the Weierstrass approximation theorem, Compactness in metric spaces, the Ascoli-Arzela theorem.
2. Lebesgue integral: sigma-algebras, measurable functions, measure, integrable functions, *Lp*-spaces, modes of convergence, decomposition of measures (Radon-Nikodym), Generation of measures (Lebesgue, Lebesgue-Stieljes).
3. Further topics in Real Analysis: Product measures (Tonelli, Fubini), Differentiation, functions of bounded variation, Approximation via convolutions.

Course Content:

1. Basic topological properties of Euclidean spaces, Algebra of sets, Cantor and Borel sets.
2. **Lebesgue Measure:** Outer measure, measurable sets and Lebesgue measure, non-measurable set, measurable functions, Littlewood’s three principles, Egoroff’s theorem.
3. **Lebesgue Integral:** Lebesgue integral of a bounded function, integral of a non-negative function, the general Lebesgue integral, convergence in measure.

4. **Differentiation and Integration:** Differentiation of monotone functions, function of bounded variation, differentiation of an integral, absolute continuity.
5. **The Classical Banach Spaces:** L^p-spaces, Holder inequality and Minkowski's inequality, convergence and completeness.
6. **General Measure and Integration Theory:** Introduction to abstract measure, signed measure, Hohn decomposition theorem, product measure, Fubini's theorem.

Learning Outcomes:

At the end of this course, students will be able to appreciate how ideas from deferent areas of mathematics combine to produce new tools that are more powerful than would otherwise be possible, and they will realize how real function theory underpins functional analysis as well as modern analysis. Students will have developed

1. mathematical intuition and problem-solving capabilities;
2. understanding of which tool is appropriate to tackle which problem;
3. ability to find information through tools like the world-wide web to solve problems;
4. ability to use computers to illustrate arguments;
5. competency in mathematical presentation, and written and verbal skills.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. H. L. Royden, Real Analysis, Pearson
2. P. R. Halmos, Measure Theory, Springer
3. Robert Bartle, The elements of integration and Lebesgue measure, Wiley Classics Library
4. Gerald Folland, Real Analysis, Modern Techniques and Their Application, Wiley
5. A. N. Kolmogorov and S. V. Fomin, Elements of the theory of functions and functional analysis, Dover books
6. E. H. Lieb and M. Loss, Analysis, Graduate Studies in Mathematics, AMS
7. Walter Rudin, Principles of Mathematical Analysis, McGraw Hill

MTMS 505: COMPLEX FUNCTION THEORY **Credits: 4**

Course Contents:

1. **Convergence:** Uniform convergence of power series. Weierstras's theorem. Absolute and uniform convergence of infinite products.
2. **Analytic Functions:** Open mapping theorem, Maximum modulus principle, Convex functions, Hadamard three-circles theorem. Caratheodory's inequality. Theorems of Poisson, Jensen, Borel and Caratheodory. Space of analytic functions.
3. **Harmonic Functions:** Basic properties, Mean value theorem. Maximum modulus principle, Poisson's Kernel, Harmonic functions of a disc. Dirichlet problem on a disc.

4. **Power Series:** Basic Properties, Relations among power series, Fourier series and Dirichlet series. Sufficient conditions of regular and singular points. Theorems of Hurwitz, Vitali and Montel.
5. **Entire Functions:** Basic properties, Order, type. Growth properties of entire functions with their Zeros. Jensen's Inequality for entire function. Expression of order and type in terms of Taylor coefficients. Hadamard product of entire functions, order and type.
6. **The Space $\mathbf{C}^n$**: Introduction to the theory of analytic function of several complex variables. Formal power series about z in $\mathbf{C}^n$, Poly-cylinder, Distinguished boundary. Reinhardt domain, analytic function. Complex holomorphic functions. Taylor series expansion. Cauchy's integral.

Rationale, Course Objectives and Learning Outcomes: The course instructor will provide the details.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. A. I. Markushevich, Theory of functions of a complex variable. Volume I, & Vol. II. Prentice-Hall, INC. N.J
2. Ralph Philip Boas, Entire Functions, Academic Press Inc. New York
3. Salomon Bochner and William Ted Martin, Several Complex variables
4. L. I. Ronkin, Theory of entire functions of several complex variables

MTMS 506: GENERAL TOPOLOGY **Credits: 4**

Rationale:

Geometry has grown out of efforts to understand the world around us, and has been a central part of mathematics from the ancient times to the present. Topology has been designed to describe, quantify, and compare shapes of complex objects. Together, geometry and topology provide a very powerful set of mathematical tools that is of great importance in mathematics and its applications. This module will introduce the students to the mathematical foundation of modern geometry based on the notion of distance. We will study metric spaces and their transformations. Through examples, we will demonstrate how a choice of distance determines shapes, and will discuss the main types of geometries. An important part of the course will be the study of continuous maps of spaces. A proper context for the general discussion of continuity is the topological space, and the students will be guided through the foundations of topology. Geometry and topology are actively researched by mathematicians and we shall indicate the most exciting areas for further study.

***Course Objectives*:**

The main objective of this topic is to compare several notions that describe convergence in topological spaces. The objectives of this course are to

1. introduce students to basic topological concepts such as weak and strong topologies, subspaces, mapping, and quotient spaces

2. introduce student how filters and nets are converging in a topological space
3. demonstrate students to the concept of Separation axioms and proof of Urysohm's Lemma. To give the idea how a topological space depends upon the distribution of open sets in the space and introduce the connection between different spaces such as regular spaces, completely regular spaces, and normal spaces
4. introduce student the concept of compactness by describing generalization of finiteness and Heine-Borel theorem to demonstrate notions of compactness and various compactification constructions
5. introduce the notion of Metrization and some applications of Baire theorem in functional analysis
6. introduce the notion ofdifferent types of connected spaces and the relation between Pathwise and local connectedness
7. introduce to the concept of uniform topological space and metrizablespace,and their relation.
8. introduce to the concept of Functional spaces and establish a relation between point-wise and uniform convergences
9. introduce students to the notion of Commutative Topological Groups, Bases, Subgroups and Quotient groups, completion of topological groups, continuous homomorphisms, Groups of functions
10. develop the student's ability to handle abstract ideas in topology to understand real world applications

Course Content:

1. **Basic Topological Concepts:** Bases and subbases, subspaces, continuous maps, weak and strange topologies, quotient spaces.
2. **Product Spaces**.
3. **Convergence:** nets and filters.
4. Separation axioms, regular spaces, completely regular spaces, Normal spaces, Urysohm's lemma, Characterization of Normality.
5. Countability properties.
6. Compact and locally compact spaces, compactification.
7. Metrization; Baire theorem.
8. **Connected Spaces:** Pathwise and local connectedness.
9. Uniform spaces, uniformizability and uniform metrizability.
10. **Function Spaces:** Pointwise and uniform convergence, compact open topology.
11. Commutative Topological Groups, Elementary considerations, Bases, Subgroups and Quotient groups, completion of topological groups, continuous homomorphisms, Groups of functions, uniformities and metrization.

***Learning Outcomes*:**

Upon successful completion of this course, the student will be able to

1. understand the proofs of convergence of filters and nets. Also, Students will learn that convergence of filters and nets are special cases of F-convergence.

2. learn Separation axioms and prove Urysohm's Lemma. Students will learn the connection between different spaces such as regular spaces, completely regular spaces, and normal spaces. Students will also understand the characterization of normality.
3. determine for a given topology which countability and separation properties it has.
4. understand to work with various notions of compactness and be familiar with various compactification constructions.
5. learn about Metrization and they will learn some applications of Baire theorem in functional analysis.
6. learn about connected spaces. They will also understand the relation between Pathwise and local connectedness.
7. study the important relation:"If a uniformity is metrizable, so is the uniform topology it generates. In the opposite direction, metrizability of the uniform topology does not imply that the uniformity itself is metrizable".
8. gain knowledge of Functional spaces and establish a relation between point-wise and uniform convergences. They will able to distinguish uniform and point-wise convergences.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. G. F. Simmons, Introduction to Topology and Modern Analysis
2. J. Kelly, General Topology.
3. Munkres. James, Topology. Pearson
4. James Dugundji, Topology, William C Brown Pub
5. S. Lipschutz, General Topology.

MTMS 507: FUNCTIONAL ANALYSIS **Credits: 4**

Rationale:

This course will cover the foundations of functional analysis in the context of topological linear spaces and normed linear spaces. This course is a natural follow of the course Topology; while the main focus of Convex sets and hyperplanes, seminorms, locally convex spaces, weak topology, compact convex sets, duality in Banach spaces. Then the linear analysis is on Hilbert spaces with its rich geometrical structures will work with normed linear spaces. The Big Theorems (Uniform Boundedness, Open Mapping and Closed Graph) will be presented and several applications will be analyzed. The important notion of duality will be developed in Banach and Hilbert spaces and an introduction to spectral theory for compact operators will be given. Moreover, Bilinear and quadratic forms, symmetric operators, normal and self-adjoint operators and spectral analysis in Hilbert Spaces for bounded self-adjoint operators and unbounded self-adjoint operators will be analyzed. Despite working in this more general framework many results on Nonlinear Compact Operators and Monotonicity will be re-introduced in this course in more general form. For example, Banach Fixed point theorem with applications, Schauder fixed point theorem, Frechet derivative, Newton's method for nonlinear operators, positive and monotone operators will be introduced.

***Course Objectives*:**

Upon completion of this course, students will explore the followings:

1. Facility with the main, big theorems of functional analysis.
2. Learn the fundamental concepts of Topological Linear Spaces and study of the properties of bounded linear maps between topological linear spaces of various kinds.
3. Ability to use duality in various contexts and theoretical results from the course in concrete situations.
4. Capacity to work with families of applications appearing in the course, particularly specific calculations needed in the context of famous theorem.
5. Be able to produce examples and counter examples illustrating the mathematical concepts presented in the course.
6. Understand the statements and proofs of important theorems and be able to explain the key steps in proofs, sometimes with variation.

Course Content:

1. **Topological Linear Spaces:** Convex sets and hyperplanes, seminorms, locally convex spaces, weak topology, compact convex sets, duality in Banach spaces.
2. **Linear Operators:** Continuity and boundedness, fundamental properties of bounded operators, uniform boundedness principle, conjugate of bounded linear operators, adjoint operator and its duality, bounded linear operators in Hilbert spaces, unbounded linear operator.
3. **Spectral Analysis of Linear Operators**: Spectrum and the resolvent operator, spectrum of a bounded linear operator, compact operators.
4. **Spectral Analysis in Hilbert Spaces:** Bilinear and quadratic forms, symmetric operators, normal and self-adjoint operators, the spectral theorem for bounded self-adjoint operators, unbounded self-adjoint operators.
5. **Nonlinear Compact Operators and Monotonicity:** Banach Fixed point theorem with applications, Schauder fixed point theorem, Frechet derivative, Newton’s method for nonlinear operators, positive and monotone operators.

***Learning Outcomes*:**

On successful completion of this course unit students will be able to

1. know and use the properties of topological linear spaces. Also, explain the concepts of Convex sets and hyperplanes, seminorms, locally convex spaces, weak topology, compact convex sets, duality in Banach spaces.
2. get familiar with the Linear Operators, in particular Continuity and boundedness, fundamental properties of bounded operators, uniform boundedness principle, Open Mapping and Closed Graph theorem, conjugate of bounded linear operators, adjoint operator and its duality, bounded linear operators in Hilbert spaces, unbounded linear operator.
3. understand the concept of spectral analysis of linear operators and be able to analyze the necessary problems and theorems on the spectrum and the resolvent operator, spectrum of a bounded linear operator and compact operators.

4. get familiar of spectral analysis in Hilbert Spaces and describe the basic terminologies appeared in bilinear and quadratic forms, symmetric operators, normal and self-adjoint operators, the spectral theorem for bounded self-adjoint operators, unbounded self-adjoint operators.
5. learn on Nonlinear Compact Operators and Monotonicity and deal with the Fixed point theorems and to recognize their applications.
6. compute Frechet derivative and apply Newton's method for nonlinear operators, positive and monotone operators.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. E. Taylor, Introduction to Functional analysis, John Wiley & Sons, Inc.
2. V. Hutson and J. S. Pym, Applications of Functional Analysis and Operator theory, Academic press
3. W. Rudin, Functional Analysis, McGraw-Hill, Inc. International
4. N. Dunford and J Schwartz, Linear operators, General Theory. Wiley

MTMS 508: LIE GROUPS AND LIE ALGEBRAS **Credits: 4**

Rationale:

Lie groups are continuous groups of symmetries, like the group of rotations of n-dimensional space or the group of invertible n-by-n matrices. In studying such groups we can use tools from calculus to linearize our problems, which leads us to the notion of a Lie algebra: a vector space with an antisymmetric product associated to any Lie group, which remembers everything about its algebraic structure. For example, the Lie algebra associated with the group of rotations of 3-space is just 3-dimensional Euclidean space with (twice) the vector cross product.

Lie algebras appear in mathematics in many ways.

i. They represent the local structure of Lie groups, groups with a differentiable structure.
ii. They represent infinitesimal actions on vector spaces, actions satisfying rules like the Leibniz rule $d(fg) = f\,dg + g\,df$.
iii. They represent the non-commutativity of an associative algebra. We will be studying Lie algebras from several points of view: algebraic; combinatorial; and geometric.

***Relation to other mathematics courses*:**

The theory of Lie groups and their representations is one of the glories of 20th century mathematics. Beginning with work of Lie and Killing in the 19th century, its outline began to take modern form in the1950s through work of Cartan, Chevalley, Weyl, and others; and since then it has become the foundation of an enormous body of work (classification of finite groups, automorphic forms, etc) in mathematics and physics (relativity, quantum mechanics, Yang-Mills theory, 'standard model', etc).

Students are expected to a have an undergraduate level background in group theory, ring theory, linear algebra and analysis. Basic topology is helpful for understanding the connection with Lie groups, and more advanced linear algebra (tensor products, symmetric products, etc.) is very useful, but neither is a prerequisite for the course.

***Course Objectives*:**

This course is divided into two halves. In the first half we introduce the notion of a closed linear group and the relationship between a closed linear group and its linear Lie algebra which will serve you well in later part of the course. We prove that any closed linear group becomes a Lie group. In the second half of the course, we turn our attention to the connection between Lie algebras and Lie groups. This will involve some ideas from geometry (manifolds and tangent spaces).

At the title suggests, this is a first course in the theory of Lie groups and Lie algebras. We focus on the so-called matrix Lie groups since this allows us to cover the most common examples of Lie groups in the most direct manner and with the minimum amount of background knowledge. We mention the more general concept of a general Lie algebra of a Lie group, but do not spend much time working in this generality. After some motivating examples involving quaternions, rotations and reflections, we give the definition of a matrix Lie group and discuss the well-studied examples, including the classical Lie groups. We then study the topology of Lie groups, their maximal tori, and their centres. We conclude with a discussion of differential (adjoint) representation.

Course Content:

1. **Manifolds:** Some basic notation and terminology, Manifolds and differentiable manifold; some important results on manifolds.
2. **Closed Linear Groups:** Topological groups, Lie Algebras of the closed linear groups.
3. **The Exponential of a Matrix:** Convergent power series, matrix power series.
4. **Lie Groups and Lie Algebras:** Lie groups, A matrix representation of the topological group R. Linear Lie groups, Lie subgroups.
5. **Lie Algebras:** The exponential map of a Lie group, General Lie algebras.
6. **Analyticity of Lie Groups:** The Lie algebra defined by a Lie group, Local coordinates, Abelian and reductive Lie groups.
7. **Correspondence between Lie Groups and Lie Algebras:** Tangent space and vector fields, Invariant vector fields, Invariant differential operators, One-parameter subgroups, Differential representation.

Learning Outcomes:

Students will have developed

1. mathematical intuition and problem-solving capabilities;
2. understanding of which tool is appropriate to tackle which problem;
3. ability to find information through tools like the world-wide web to solve problems;
4. ability to use computers to illustrate arguments;
5. competency in mathematical presentation, and written and verbal skills.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. Anthony W. Knapp. Lie Groups Beyond an Introduction, Birkhäuser
2. F. W. Warner. Foundations of Differentiable Manifolds and Lie groups, Springer
3. M. Spivak. Calculus on Manifolds, Westview Press
4. A. A. Sagle and R. E. Walde, Introduction to Lie Groups and Lie Algebras, Academic Press

MTMS 509: FUZZY MATHEMATICAL STRUCTURES **Credits: 4**

Course Content:

1. **Basics of Fuzzy Sets:** Constructing Fuzzy sets, Operations on Fuzzy sets, t-norm and s-norm, α-Cuts, Extension principle, Measurement of fuzziness, Fuzzy relations, Fuzzy similarity, Fuzzy ordering, Pattern classification based on fuzzy relations, Fuzzy relational equations.
2. **Fuzzy Numbers:** Representation theorems, Interval analysis, Arithmetic operations, Applications.
3. **Fuzzy Logic:** Multi-valued logics, Fuzzy propositions, Fuzzy quantifiers, Linguistic hedges, Approximate reasoning.
4. **Fuzzy Topological Spaces:** Fuzzy topologies, F-Continuous functions, Fuzzy metric spaces, Fuzzy neighborhood spaces, Fuzzy convergence, Fuzzy compact spaces, Fuzzy connectedness, Fuzzy components.
5. **Fuzzy Algebra:** Fuzzy substructures of algebraic structures, Fuzzy monoids, and automata theory, Fuzzy subgroups and pattern recognition, Free fuzzy monoids, and coding theory.

****Rationale, Course Objectives and Learning Outcomes:*** The course instructor will provide the details.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. J. N. Mordeson & P. S. Nair. Fuzzy mathematics: An introduction for Engineers and Scientists, Physica-Verlag Heidelberg
2. G. J. Klair, U. S. Clair & B. Yuan. Fuzzy Set Theory: Foundations and applications, Prentice Hall
3. R. Lowen, Fuzzy Set Theory, Springer
4. Liu Ying-Ming & Luo Mao-Kang, Fuzzy Topology, River Edge, NJ: World Scientific Pub.
5. H.J. Zimmermann, Fuzzy set Theory and its Applications, Springer

MTMS 510: SPECIAL TOPICS **Credits: 4**

Detailed syllabus will be given by the Department.

MTMS 511: DIFFERENTIAL AND INTEGRAL EQUATIONS **Credits: 4**

Rationale:

This course is intended to develop rigorous practical and analytic skills in differential and integral equations (DIE). It is intended to illustrate various applications of differential and integral equation to technical problems as well. The laws of nature are expressed as differential and integral equations. Scientists and engineers must know how to model the world in terms of differential and integral equations, and how to solve those equations and interpret and analyze the solutions. This course focuses on theoretical aspects of linear and nonlinear differential and integral equations and their applications in science and engineering. More details are given in the course goals below.

***Course Objectives*:**

1. To give knowledge on some basic mathematical analysis of solutions of differential and integral equations.
2. To know how to interpret the solutions of DIE.
3. To learn about the application DIE to model and analyze real life problems.
4. Numerical solutions of integral equations.

Course Content:

1. **Existence and Uniqueness Theorem of Differential Equations**: Methods of successive approximations, Picard's method, Peano's existence theorem, fixed point methods, continuation of solutions, and existence theorem for vector valued IVP.
2. **Stability Analysis**: Stability analysis of linear and nonlinear differential equations, Lyapunov stability analysis.
3. **Integral Equations:** Conversions of IVP's to integral equations, existence, uniqueness and general properties of solutions of Volterra integral equations, linear and non-linear systems of VIE's resolving kernels, Fredholm theory of IE's, semi-analytic solutions of a class of integral equations of Volterra and Fredholm types.
4. **Periodic Solutions:** Periodic solutions of linear and non-linear differential and integral equations.
5. **Numerical Solutions of Integral Equations**: Introduction, collocation method, Galerkin's method, trapezoidal method, review of some articles.

***Learning Outcomes*:**

Students will be able to

1. explain the concept of differential and integral equations
2. expresses the existence-uniqueness theorem of differential and integral equations.
3. explains basic properties of solutions of DIE.
4. Obtain analytic solutions of differential equations.
5. Obtain analytic and semi-analytic solutions of a class of integral equations.

6. Gain knowledge about the applicability of differential equations and integral equations in various branches science and engineering.
7. Approximate solutions of integral equations using various schemes.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. Morris W. Hirsch, Stephen Smale, Robert L. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos
2. T. A. Burton, Stability and Periodic Solutions of Ordinary and Functional Differential Equations
3. S. L. Ross, Differential Equations
4. Fred Brauer and John A. Nohel, Ordinary differential equations
5. Rama Mohana Rao, Ordinary Differential Equations: Theory and Applications
6. Masujima, M., Weinheim, Applied Mathematical Methods of Theoretical Physics: Integral Equations and Calculus of Variations, Germany, Wiley
7. Sharma, Gupta and Agarwal, Integral equations
8. D. E. Atkinson, Numerical solutions of integral equations.

MTMS 512: OPERATIONS RESEARCH **Credits: 4**

Rationale:

Operations Research (OR), also called Management Science, is the study of scientific approaches to decision-making problems. Through mathematical modeling, it seeks to design, improve and operate complex systems in the best possible way. This is a comprehensive course covering several areas of OR. The module covers topics that include: linear programming, bounded variable simplex algorithm, transportation and assignment problem, job sequencing, network model, dynamic programming, integer programming, game theory and nonlinear programming. Analytic techniques and computer packages will be used to solve problems facing different real life application oriented decision making problems.

***Course Objectives*:**

The objectives of this course are to:

1. formulate a real-world problem as a mathematical programming model.
2. implement and solve the model using various software packages.
3. solve specialized linear programming problems like the transportation and assignment problems.
4. solve network models like the shortest path, minimum spanning tree, and maximum flow problems.
5. understand the applications of, basic methods for, and challenges in integer programming.
6. understand how to model and solve problems using dynamic programming.

7. learn optimality conditions for single- and multiple-variable unconstrained and constrained nonlinear optimization problems, and corresponding solution methodologies

Course Content:

1. **Basics of Operations Research:** Introduction, Definition, Characteristic, Necessity, Scope, Classification of problems, Types of mathematical models, Review of Linear Programming.
2. **Transportation and Assignment Problem**: Introduction, Formulation, Relationship with LP, Solution procedure and Applications.
3. **Network Models:** Network definitions, Shortest Route problem, Minimal Spanning Tree problem and Maximal-Flow problem.
4. **Integer Programming:** Introduction, Branch and Bound Algorithm, Cutting-plane Algorithm, Application.
5. **Sequencing Problem:** Sequencing problem processing *n* jobs through two machines, *n* jobs through three machines, two jobs through *m* machines, *n* jobs through *m* machines and approaches to more complex sequencing problems.
6. **Matrix Game Theory:** Introduction, Minimax-maximin pure strategies, Mixed strategies and Expected payoff, solution of *2X2* games, solution (2Xn) and (mX2) games, solution of (m x n) games by linear programming and Brown's algorithm.
7. **Dynamic Programming:** introduction, Investment Problem, Production scheduling problem, Stagecoach problem, Equipment replacement problem.
8. **Nonlinear Programming:** Introduction, Unconstrained problem, Lagrange method for equality constraint problem, Kuhn-Tucker method for inequality constraint problem and Quadratic programming problem.

***Learning Outcomes*:**

1. Students will be able to model and solve some real life oriented problems such as, transportation and assignment problem. Also they will be able to connect these problems with the network models.
2. They will be familiarized with job scheduling problems along with their solution procedures.
3. Application of bounded variable linear program will be understood in modeling several network models.
4. They will learn modeling with integer program along with the solution techniques.
5. Students will get some basic backgrounds on Dynamic programming and Game theory.
6. They will be able to solve constrained and unconstrained nonlinear optimization problems.
7. Students will be able to develop applications using the familiar software tools (EXCEL/SOLVER, LINDO, MATLAB, MATHEMATICA, etc.) to solve problems.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. Winston, Operations Research: Applications and Algorithms, Cengage Learning
2. Hiller, F. S. and L. J. Lieberman, Introduction to operation research, McGraw-Hill

3. Handy A. Taha, An Introduction to Operation Research, Pearson Education
4. Amir Beck, Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB, SIAM
5. A. Ravindran (Author), Don T. Phillips (Author) and James J. Solberg, Operations Research: Principles and Practice

MTMS 513: NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS Credits: 4

Rationale:

There are a lot of naturally occurred processes which can be described using ordinary and partial differential equations (ODEs and PDEs). A thorough knowledge of these processes are acquired solving the relevant equations. This course deals with numerical methods of various types of ordinary and partial differential equations. In particular, finite difference methods (FDMs) for linear and nonlinear ordinary differential equations as well as for elliptic, parabolic, hyperbolic partial differential equations will be discussed. Moreover, students will learn finite element methods (FEMs) in details.

Course Objectives:

1. To learn FDM for linear and nonlinear ODEs.
2. To know how to solve PDEs using FDMs.
3. To find numerical integration using FEM.
4. To provide a detailed knowledge about FEM to solve PDEs.
5. To learn how to solve eigenvalue problem using FEM.

Course Content:

Group A: Finite Element Method

1. **Introduction to FEM:** Discretization, Construction of basis functions, Numerical integration; coordinate transformation, local and global derivatives, mesh generation, h-p convergence, finite element approximation of line and double integrals.
2. **Method of Weighted Residuals:** Subdomain, Collocation, Galerkin, and Least-Squares methods, Matrix Formulation; Modified Galerkin techniques, Element/stiffness matrix.
3. **Finite Element Solution of BVP:** Outline of FE procedures for 1-D and 2-D problems (Poisson’s and Laplace’s equations), Matrix Formulation, Element concept, Assembly, Triangular, Rectangular and Quadrilateral elements (linear and quadratic elements).
4. **Variational Formulation of BVP:** Functional and Variational Calculus, Construction of Functionals, Rayleigh Ritz Method and Finite elements.

Group B: Finite Difference Method

5. **Review** of Finite difference approximation. Applications to solve eigenvalue problems and higher order BVP.

6. **Elliptic PDEs:** Difference equations for Poisson's and Laplace's Equations BVP, matrix formulation, Convergence by iterative methods, and error analysis.
7. **Parabolic Problems:** Derivation of difference formulas for IBVP, Matrix formulation, Heat Equation, Forward, Backward and Crank-Nicolson Methods, Difference methods in 2-space dimensions, ADI method, Stability and error analysis.
8. **Hyperbolic Problems:** Difference methods for a scalar IVP and IBVP, Lax-Wendroff and Courant-Friedrichs-Lewy explicit methods, Wendroff implicit method, Wave Equation in time dependent and two space dimension, Convergence and stability analysis.

Learning Outcomes:

1. Ability to solve linear and nonlinear ODEs employing FDM
2. Analyze PDEs using relevant FDMs.
3. Find numerical integration using FEM.
4. Apply FEM in solving PDEs.
5. Solve eigenvalue problem utilizing FEM.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. P. E. Lewis and J. P Ward, The finite element method: Principles and Application, Addison Wesley
2. O. C. Zienkiewicz and K. Morgan, Finite Elements and approximations, John Wiley and Sons
3. M. A. Celia and W. G. Gray, Numerical Methods for Differential Equations, Prentice-Hall Int. Inc.
4. G. D. Smith, Numerical solution of Partial differential equations, Clarendon press, Oxford
5. Paul D. and David Zachmann, Applied Partial differential Equations, Harper & Row Publications, New York
6. A. R. Mitchell and R. Wait, Finite Element Method in Partial Differential Equations, John Wiley & Sons Ltd
7. A. R. Mitchell and D. F. Griffiths, The Finite Difference Method in Partial Differential Equations, Wiley

MTMS 514: GEOMETRY OF DIFFERENTIAL MANIFOLDS **Credits: 4**

Rationale:

Geometry of Differential Manifolds is based on three dimensional basic vector geometry of curves and surfaces with calculus. Understanding of this course will precede students to learn other areas of mathematics such as Differentiable Manifolds, Riemannian Manifolds, Theory of Relativity and cosmology etc. Upon the successful completion of this course students will able to :

i. Apply the concepts of surfaces to find which surface are minimal surfaces and also to know Weingarten, Gauss and Codazzi equations, Theorema Egreegium, fundamental theorem of surface theory etc.

ii. Students will know the concepts of developable surfaces, ruled surfaces, Gaussian curvature, Geodesics, Geodesic curvature, Liouvilles formula, Clairaut's theorem, Bonnet's formula and Gauss-Bonnet theorem.

iii. Students will learn about Conformal, isometric and geodesic mapping, Tissot's theorem, Theory of differential functions, charts, atlases, differentiable manifolds, smooth map on Manifolds, Tangent space, Tangent bundles, C^{∞} -vector fields and Lie brackets of vector fields on Manifolds, φ - related vector fields.

Course Objectives:

1. To give knowledge on mathematical concepts of space curve and different types surfaces, this course is very much useful.
2. Students will know the concepts of geodesic curvature κ_g and its formulae, Liouville's formula, geodesic on a surface of revolution, Clairaut's theorem, Bonnet's formula, geodesics on Liouville surface, Gauss-Bonnet theorem.
3. Students will learn about Manifold structure on a topological space, C^{∞} -vector fields on manifolds etc.

Course Content:

1. **Surfaces and Properties of Surface:** Minimal surfaces, theorem of minimal surfaces, general solution of the natural equations, Riccati equation and its solution, equation of Weingarten, Gauss and Codazzi and their applications, Theoema Egreegium, fundamental theorem of surface theory.
2. **Developable and Ruled Surfaces:** Envelop, characteristic, edge of regression, developable surface, property of lines of curvature on developable, ruled surface, fundamental coefficients and Gaussian curvature for ruled surface, tangent plane to a ruled surface.
3. **Geodesics on a Surface:** Geodesics, differential equation of geodesics, geodesics on plane, surface, sphere, right circular cone, right helicoid, cylinder, torus etc., geodesic curvature κ_g and its formulae, Liouville's formula, geodesic on a surface of revolution. Clairaut's theorem, Bonnet's formula, geodesics on Liouville surface, Gauss-Bonnet theorem, torsion of a geodesic, geodesic parallel.
4. **Mapping of Surfaces:** Mapping, homeomorphism, isometric lines and correspondence, Minding theorem, conformal, isometric and geodesic mapping, Tissot's theorem.
5. **Differentiable Manifolds:** Theory of differentiable functions, coordinate functions, charts and atlases, complete, compatiablity, differentiable structures, differentiable manifolds, local representation of a function for charts, induced topology on a manifolds.
6. **Topology of a Manifold:** Manifold structure on a topological space, properties of induced topology, topological restrictions on manifolds.
7. **Differentiation on a Manifolds:** Partial differentiations, equivalence relation and class, smooth map on manifolds, tangent space, tangent bundles, tensor and exterior bundles, tangent map on manifolds.

8. **Vector Fields on a Manifolds:** C^∞-vector fields on manifolds, coordinates of vector fields, set of vector fields, theorem on vector fields and its coordinates, Lie brackets of vector fields and properties of φ -related vector fields.

***Learning Outcomes*:**

1. Apply Gauss and Weingarten equations to find out Theorema Egreegium and Codazzi's equations.
2. Describe Riccati equation and its solution with some problem.
3. Know how to check developable surfaces and how to find Gaussian Curvature and which surface is ruled surface or skew.
4. How to find the geodesics for surfaces of plane, sphere, right circular cone, right helicoid, cylinder and torus etc.
5. Illustrate different types of mapping and their properties and proof Tissof's theorem by using non-conformal mapping.
6. Know about compatiability using composite of two charts.
7. Illustrate the differential structure of manifold with C^∞-function and topology.
8. Gather Knowledge about smooth map, tangent space, tangent bundles, structure of tangent space map on manifolds.
9. Apply the concepts of φ -related vector fields to find a lemma and by using this lemma a proposition of Lie brackets of vector fields is proved.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. C. E. Weatherburn, Differential Geometry of Three Dimensions, Cambridge University press, London
2. D. J. Struik, Lectures on Classical Differential Geometry, Addison–Wesley Publishing Company, Inc. U.S.A.
3. F. Brickell and R. S. Clark, Differentiable Manifolds: An Introduction, Van Nostrand Reinhold Company, London
4. F. W. Warner, Foundations of Differentiable Manifolds and Lie groups, Scott, Foresman and Company, Glenview, Illiniois, London
5. S. C. Mital and D. C. Agarwal, Differential Geometry, Krishna Prakashan Mandir, India

MTMS 515: DYNAMICAL SYSTEMS **Credits: 4**

Rationale:

Dynamics is the subject that deals with change, with systems that evolve in time. Whether the system in question settles down to equilibrium, keep repeating in cycles, or does something more complicated, it is dynamics that we use to analyze the behavior in various places of science.

***Course Objectives*:**

At the end of the year students should be able to know:

1. The qualitative properties of dynamics and to understand asymptotic behavior
2. To classify equilibria by their stability, invariant manifolds and topological types
3. Identify fundamental differences between linear and nonlinear dynamical systems
4. Construct and interpret phase portraits of maps and flows
5. Identify fixed points and periodic points and determine their stability
6. How qualitative structure of the flow can change as parameters are varied
7. Unpredictable long-term behavior in a deterministic dynamical system
8. Characterization and measurements of chaos such as sensitive dependence on initial conditions and Lyapunov exponents
9. Use fractals to predict or analyze various biological processes or phenomena

Course Content:

1. **Basic Concepts:** Basic definitions & examples, Phase space, phase portrait, discrete dynamics, continuous dynamics, conjugacy, fixed and periodic points, hyperbolic points, hyperbolic dynamics, conjugacy, Cantor set.
2. **Chaotic Dynamical Systems**: Definitions of chaos, sensitive dependence on initial conditions, orbit structure, Cantor set, basin of attractor & repeller, strange attractors, lyapunov exponents.
3. **Discrete Dynamical Systems**: One parameter family of maps, contractions & fixed points, stability of fixed points, family of logistic map, tent map, linear maps, iterative map, quadratic family, Smale horseshoe map, expanding map.
4. **Differential Dynamical Systems**: One & two dimensional linear & nonlinear differential equations, sinks, source & saddles, stability, population models, Lotka-Volterra models, Henon map, hyperbolic fixed point, manifold and sub-manifold, stable and unstable subspaces, stable manifold theorem, Hartman-Grobman theorem, Hadamard-Perron theorem, Smale theorem.
5. **Bifurcations**: Bifurcations, bifurcation points, Saddle-node, period-doubling, pitchfork, trancritical bifurcation, bifurcation diagram.
6. **Symbolic Dynamical Systems**: Sequence spaces, shift map, symbolic dynamics, subshift of finite type.
7. **Fractal**: Fractal and fractal dimension.

**** Learning Outcomes:*** The course instructor will provide the details.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. S. H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, And Engineering, Westview press
2. R. L. Devaney, A First course in chaotic dynamical systems, Westview Press
3. R. A. Holmgren, A first course in discrete dynamical systems, Springer

4. K. L. Alligood, T. D. Sauer, J. A. Yorke, Chaos: An introduction to dynamical systems.
5. R. C. Robinson, Dynamical Systems: stability, Symbolic Dynamics, and Chaos. CRC Press
6. A. Katok and B. Hasselblatt, Introduction to Modern Theory of Dynamical Systems, CUP, Cambridge

MTMS 516: MATHEMATICAL BIOLOGY **Credits: 4**

Rationale:

To provide students with the mathematical tools used to study and solve a variety of problems in biology at different scales. Mathematical Biology is one of the most rapidly growing and exciting areas of Applied Mathematics. This is because recently developed experimental techniques in the biological sciences, are generating an unprecedented amount of quantitative data.

Course Objectives:

By the end of the module the student should be able to

1. analyze simple models of biological phenomena using mathematics
2. reproduce models and fundamental results of biological systems
3. introduce the student to advanced mathematical modeling in the Life Sciences
4. apply methods in the module to new problems inside the scope of Mathematical Biology
5. explore methods for solving the models and discuss the implications of the predictions.

Course Content:

1. **Single Species Continuous Models:** Introduction to linear and nonlinear population models, Sharpe-Lotka age-dependent population model, Gurtin-MaCamy age-dependent population model, stability.
2. **Multi Species Continuous Models:** Two species linear and nonlinear population models, multi-species models, stability.
3. **Microbial Population Models**: Microbial population, chemostat, growth of microbial populations, dynamics of microbial competition, stability.
4. **Dynamics of Infectious Diseases:** Virus, immunity, cells, epidemic models, dynamics of infectious diseases, AIDS/HIV models, dynamics of hepatitis B virus, age-dependent epidemic model, control of an epidemic, drug therapy, vaccination effects, treatment of HIV, CTL responses, immune response dynamics.
5. **Dynamics with Diffusion**: Diffusion equation, single and multi-species diffusion models, competition model with diffusion, epidemic model with diffusion.
6. **Stochastic Model**: Concepts in probability, stochastic Processes, Brownian motion, martingales, stochastic linear and nonlinear models of population.
7. **Applications**: Glucose concentration in blood, heart beat model, tumor growth model, blood cell growth etc.

Learning Outcomes:

After completing this course, the students will be able to understand

1. the applications of ODE models in a variety of biological systems,
2. making the student aware how to choose and use different modeling techniques in different areas
3. reaction-diffusion equations and their applications in biology
4. introduce the connections between biological questions and mathematical concepts
5. develop the mathematics of dynamical systems, linear algebra, differential equations and difference equations through modeling biological systems.
6. explore the utility of using mathematical tools to understand the properties and behavior of biological systems.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. F. Brauer & C. Castillo-Chavez, Mathematical models in population biology and epidemiology, Springer-Verlag, New York
2. J. D. Murray, Mathematical Biology, Springer
3. Leah Edelstein-Keshet, Mathematical Models in Biology
4. H. L. Smith & P. Waltman, Theory of Chemostat, CUP
5. M. A. Nowak & R. M. May, Virus Dynamics, Mathematical Principles of Immunology and Virology

MTMS 517: OPERATIONS MANAGEMENT **Credits: 4**

Rationale:

Operations activities, such as forecasting, choosing a location for an office or plant, allocating resources, designing products and services, scheduling activities, and assuring and improving quality are core activities and often strategic issues in business organizations. Production Management or Operations Management is the management of systems or processes that create goods and/or services. The material in this course is intended as an introduction to the field of operations management. The field of operations management is dynamic, and very much a part of the good things that are happening in business organizations. Much of what the students learn will have practical application.

***Course Objectives*:**

1. To give knowledge on the ways to manage the business organization efficiently.
2. Students will be able to learn the formulating procedure of different types of management tools.
3. It will help the students to apply the knowledge gather from this course in real life problems.

Course Content:

1. **Introduction:** Introduction to Operations Management, The scope of OM, OM and decision making, Productivity, Product mix, Strategy, Competitiveness.

2. **Capacity Planning:** Strategic capacity decision, Strategy formulation, Defining and measuring capacity, Evaluating capacity alternatives.
3. **Quality Control:** Management of quality, Statistical process control, Variations and control, Control charts, Process capability, Improving process capability, Capability analysis.
4. **Forecasting:** Features common to all forecasts, Elements of good forecast, Steps in the forecasting process, Accuracy and control of forecasting, Applications, Forecasting models.
5. **Inventory Control:** Nature and importance of inventories, Introduction to basic inventory models (Economic order quantity (EOQ) model, EPQ model, Fixed order interval model, Single period model.
6. **Scheduling:** Scheduling in high-volume systems, intermediate-volume systems, low-volume systems, Scheduling methods of Linear Programming, Scheduling jobs through two work centers, Minimizing scheduling difficulties, Scheduling the work force.
7. **Simulation:** Basic terminology of simulation, Steps in simulation process, Application of simulation, Simulations with random variables, Advantage and limitations of using simulations.
8. **Project Management:** Behavioral aspect of project management, Key decisions in project management, PERT (program evaluation and review technique), CPM (critical path method), Deterministic time estimates, Probabilistic time estimates, Applications.

***Learning Outcomes*:**

After completing this course, the students will be able to an expert in the following areas: in product and service design, process selection, technology selection, design of work systems, location planning, facility planning, quality improvement of goods and services, forecasting, capacity planning, scheduling, managing inventory, assuring quality, motivating employees, and deciding where to locate facilities.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. William J. Stevenson, Operations Management.
2. Wayne L. Winston, Operations Research
3. Hillier and Lieberman, Introduction to Operations Research
4. Hira & Gupta, Problems in Operations Research
5. Turban & Merideth, Fundamentals of Management Science.

MTMS 518: ASYMPTOTIC AND PERTURBATION THEORY **Credits: 4**

Rationale:

The course provides theoretical representations of solutions of many science and engineering problems. This course covers the order of functions, asymptotic expansions of integrals, roots of algebraic equations and solutions of ordinary differential equations for small and large parameters as well as composite solutions.

Course Objectives:

1. To understand the concept of order.
2. To understand the asymptotic expansion of integrals.
3. To understand the solutions of algebraic equations.
4. To understand the asymptotic solutions of differential equations.
5. To understand about the composite solutions.

Course Content:

1. **Asymptotic Expansions:** Definitions of asymptotic sequences, expansions, and series. Order symbols and Gauge functions, convergent versus asymptotic series, Uniqueness of asymptotic series, Elementary operations on asymptotic expansions.
2. **Expansion of Integrals:** Integration by parts, Laplace's method and Watson's Lemma, method of steepest descents, method of stationary phase.
3. Transform integrals and their asymptotic evaluation.
4. **Differential Equations:** Singularities and asymptotic methods of solutions with a large or small parameter (WKB method). Transition points.
5. **Asymptotic Matching:** Use of asymptotic matching for expansion of integrals.
6. Straightforward expansions of sources of non-uniformity.
7. The method of strained co-ordinates.
8. The method of matched and composite asymptotic expansions.

Learning Outcomes:

Upon completion of this course, students will be able to

1. Gain knowledge about order of a function, gauge functions and asymptotic sequences.
2. Determine expansion of integrals using different methods.
3. Find the solutions of differential equations for a large or small parameter.
4. Have expansion of integrals using asymptotic matching.
5. Use matched and composite asymptotic expansions.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

References:

1. J. D. Murray, Asymptotic Analysis, Springer
2. F. W. J. Olver, Asymptotic and Special Functions, A K Peters/CRC Press
3. A. H. Nayfeh, Perturbation Methods, Wiley-VCH
4. Carl M. Bender, Steven A. Orszag, Advanced Mathematical for scientists and engineers, McGraw-Hill Book Company

MTMS 519: QUANTITATIVE FINANCIAL RISK MANAGEMENT Credits: 4

Course Objectives: It is a follow up course with prerequisites of 'Stochastic Calculus' and 'Introduction to Mathematical Finance'. The idea and concept of risk management in Banks, Insurance and other financial institutions are covered from stochastic modelling perspectives. Stock markets volatility modeling and stock derivative price modelling are good part of the course. Back-testing risk measures for stock return and other risk measures for derivative investments are covered. In addition to celebrated Black and Scholes model several non-normal derivative pricing models are covered.

Course Content:

Risk Management, Financial Returns and the Dynamics

Risk Management and the Firm, A Brief Taxonomy of Risks, Stylized Facts of Asset Returns, Diffusion processes, Jump processes and some GARCH processes for risk management; Empirical Exercises, chapter References for further reading

1. **Volatility Modeling:** Simple Variance Forecasting, The GARCH Variance Model, Extensions to the GARCH Model, Maximum Likelihood Estimation, Variance Model Evaluation, Using Intraday Information, Empirical Exercises, Chapter References for further reading
2. **Correlation Modeling in Finance:** Value at Risk (VaR) for Simple Portfolios, Portfolio Variance, Modeling Conditional Covariances, Modeling Conditional Correlations, Quasi-Maximum Likelihood Estimation, Realized and RangeBased Covariance, VaR from Logarithmic versus Arithmetic Returns, Empirical Exercises, Chapter References for further reading.
3. **Modeling the Conditional Distribution in Finance:** Visualizing Non-Normality, The Standardized t(d) Distribution, The Cornish-Fisher Approximation to VaR, Extreme Value Theory (EVT), The Expected Shortfall (ES) Risk Measure, Empirical Exercises, Chapter References for further reading**.**
4. **Simulation-Based Methods:** Historical Simulation (HS), Weighted Historical Simulation (WHS), Multi-Period Risk Calculations, Monte Carlo Simulation (MCS), Filtered Historical Simulation (FHS), Empirical Exercises, estimating VaR and ES for ARCH/GARCH processes; jump-diffusion processes; fattailed levy processes; mean reverting processes. Review of Vasicek Model, exponential Vasicek model, CIR model, mean reversion +CIR combined model in order to apply risk measures. Chapter References for further reading
5. **Option Pricing:** Basic Definitions, Option Pricing under the Normal Distribution, Allowing for Skewness and Kurtosis, GARCH Option Pricing Models, Implied Volatility Function (IVF) Models, The CFG Option Pricing Formula, Empirical Exercises, Chapter References for further reading**.**
6. **Modeling Option Risk:** The Option Delta, Portfolio Risk Using Delta, The Option Gamma, Portfolio Risk Using Gamma, Portfolio Risk Using Full Valuation, A Simple Example, Pitfall in the Delta and Gamma Approaches, Empirical Exercises, Chapter References for further reading.
7. **Backtesting Risk Models:** Backtesting VaRs, Increasing the Information Set, Conditional coverage test, Unconditional coverage test, Independence test, application of backtesting to simulation based models of section 5; Empirical Exercises, Chapter References for further reading**.**

Learning Outcomes:

1. Preparing to engage in research degrees in Mathematical Finance and advanced Actuarial Science.
2. Having in-depth idea of how stock market models using Brownian motion and Brownian motion with jumps work in stochastic calculus sense.
3. Concepts of Implied volatility in derivative markets and implied volatility models for derivative pricing will help students engage in quantitative research groups of Banks and Insurance companies that deal with derivatives.
4. GARCH models for financial econometric with non-stationary distributions in stock return modeling; and risk management through Value-at-Risk (VaR) and Expected Shortfall (ES).
5. Backtesting risk measures to understand the performance of risk measures on practical data.

Evaluation: Incourse Assessment: 30 Marks. Final examination (Theory, 4 hours): 70 Marks. **Eight** questions of equal value will be set, of which any **five** are to be answered.

Reference:

1. Elements of Financial Risk Management, Peter Christoffersen, Academic Press
2. A stochastic process toolkit for Risk Management, Damiano Brigo et al
3. Quantitative Risk Management: Concepts, techniques and tools, Alexander McNeil, Princeton University Press
4. Measuring Market Risk, Kevin Dowd, Wiley

MTMS 520: SPECIAL TOPICS **Credits: 4**

Detailed syllabus will be given by the Department.

MTMS 590: MS THESIS **Credits: 8**

A student has to earn a minimum Honours CGPA as a requirement of getting entry in **Group B (Thesis Group)**. In each academic year, the Departmental Academic Committee determines the minimum CGPA. The students choose his/her supervisor among the faculties of the department subject to approval of the Academic Committee.

Each thesis student is required to work on a specific topic in different fields of mathematics and its applications, prepare a thesis report as a partial fulfillment of requirements for the degree. The thesis work may include original research, review work, and applications of mathematics and may involve fieldwork and use of technology.

Evaluation: The distribution of marks for each thesis shall be as follows:

Thesis Report	150 marks (6 credits)
Thesis Presentation	50 marks (2 credits)

MTMS 599: VIVA VOCE **Credits: 4**

Viva Voce on courses taught in the M. S. program.